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FDEM failure analysis: Bell tower of church of St. Francis of Assisi on Kaptol in Zagreb

Matej Šodan[1], Boris Trogrlić[2], Damir Foretić[3], Antonio Munjiza[4], Ivan Balić[5], Hrvoje Smoljanović[6]

[1] *Student*, University of Split, Faculty of C.Eng., Arch. & Geod., *msodan@gradst.hr*
[2] *Full Professor*, University of Split, Faculty of C.Eng., Arch. & Geod., *boris.trogrlic@gradst.hr*
[3] *Licensed Architect*, FORETIĆ I SINOVI d.o.o. Dugo Selo, *damir@foretic.hr*
[4] *Full Professor* (tenure), University of Split, Faculty of C.Eng., Arch. & Geod., *ante.munjiza@gradst.hr*
[5] *Assistant Professor*, University of Split, Faculty of C.Eng., Arch. & Geod., *ivan.balic@gradst.hr*
[6] *Assistant Professor*, University of Split, Faculty of C.Eng., Arch. & Geod., *hrvoje.smoljanovic@gradst.hr*

Abstract

This paper presents a failure analysis of the bell tower of The Church of St. Francis of Assisi on Kaptol in Zagreb on seismic action using the Finite-Discrete Element method - FDEM. The bell tower is made of masonry, and throughout history it has had a number of damages and reconstructions. It was significantly damaged in the Zagreb and Petrinja earthquakes. The analysis was performed on a simplified FDEM 2D model, and the obtained results showed a good match between the direction of cracks in the numerical model and the cracks that occurred due to these earthquakes.

Key words: belfry, FDEM analysis, masonry, earthquake

1 Introduction

The Church of St. Francis was built during the period from 1255 to 1264. In 1435 the monastery, church, bells and workshops burned down in a fire. The church was rebuilt throughout the middle of the 15th century at which time the bell is assumed to be built. Over its long history, the bell tower was damaged several times, especially in the earthquake of 1880 when it had to be removed to the height of the church cornice. It was restored by Herman Bollé to its present form. It was significantly damaged in the Zagreb and Petrinja earthquakes.

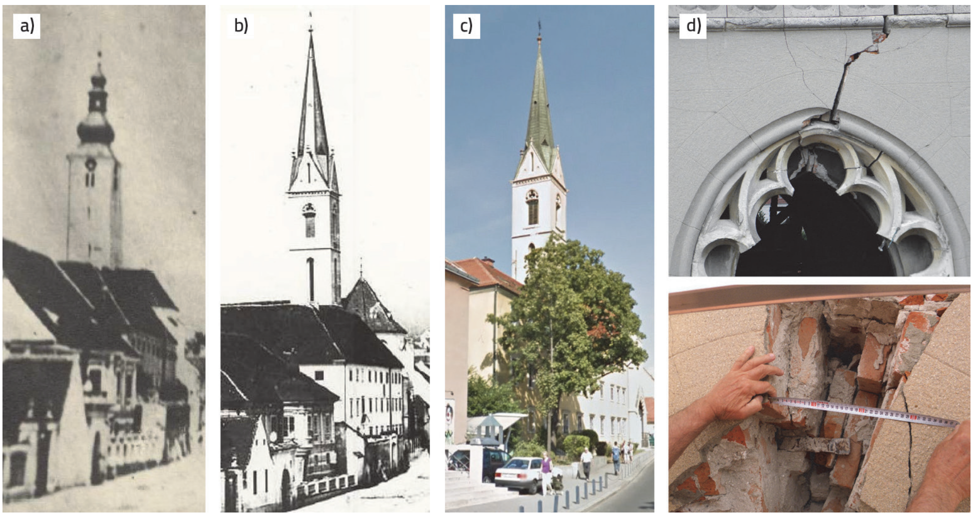

Figure 1. The bell tower of The Church of St. Francis of Assisi on Kaptol in Zagreb throughout history: a) Bell tower until 1880, b) Bell tower 1885, c) Bell tower 2020, d) Some damage of the bell tower

2 FDEM numerical modelling

Combined Finite-Discrete Element method is intended for the dynamic analysis of a large number of mutually interacting discrete elements, where the elements can fracture and fragment thus increasing the total number of discrete elements [1]. Within FDEM, each discrete element is discretised with its own finite element mesh thus enabling the deformability of discrete elements. Fracture and fragmentation processes are also implemented within the finite element mesh. The mass of discrete elements is lumped into the nodes of finite elements, while the time integration of the motion equation is applied node by node and degree-of-freedom by degree-of-freedom. This is performed in explicit form by using the central difference time integration scheme. The contact forces resulting from the interaction process between two discrete elements are determined by the numerical representation of contact impact, which is executed by employing contact detection and contact interaction procedures [1-3].

2.1 Contact detection and interaction

A contact detection algorithm is aimed at detecting the pairs of mutually contacting discrete elements and eliminate the non-contacting pairs that are far apart. Munjiza-NBS contact detection algorithm is implemented in Y code based on FDEM [2]. The total CPU time required by this algorithm to detect all contacting pairs of discrete elements is proportional to the total number of discrete elements.Once the contacting couples are detected by the contact detection algorithm, the contact interaction algorithm is applied to calculate the contact forces between them.

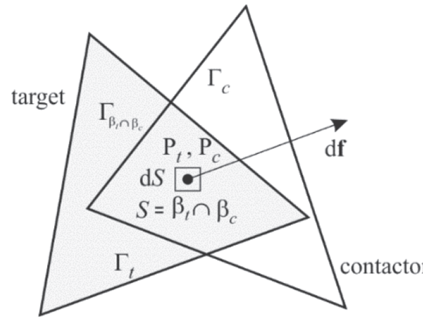

Figure 2. Contact force due to an infinitesimal overlap around points P_t and P_c

The contact interaction between the discrete elements is calculated by using the distributed potential contact force based on the penalty function method [3] which is based on the assumption that two contacting bodies, one denoted as the contactor and the other as the target, penetrate into each other thus generating the distributed contact force. As the contactor penetrates an elemental surface *dS* into the target, it generates the infinitesimal contact force given by:

$$d\mathbf{f}_c = \left[grad\phi_c \left(\mathrm{P}_c \right) - grad\phi_t \left(\mathrm{P}_t \right) \right] dS \tag{1}$$

where P_t and P_c represent the points in which the target and the contactor overlap, while ϕ is a potential field assuming the zero-equalling value on the edge and the maximum value at the centre of the discrete element.

The total contact force exerted by the target onto the contactor is obtained by the integration of the infinitesimal contact force $d\mathbf{f}_c$ over the overlap surface *S* (Fig. 2), which leads to:

$$\mathbf{f}_c = \int_{S=\beta_t \cap \beta_c} \left(grad\phi_c - grad\phi_t \right) dS \tag{2}$$

The previous equation can also be written as an integral over the line Γ of the overlapping surface as follows:

$$\mathbf{f}_c = \int_{\Gamma_{\beta_t \cap \beta c}} \mathbf{n} \left(\phi_c - \phi_t \right) d\Gamma \tag{3}$$

It should be emphasised that each triangle is considered twice, once as a contactor and once as a target. Within the contact interaction algorithm, Coulomb-type law of friction is also implemented.

2.2 Fracture and fragmentation

There are several approaches to fracture and fragmentation in the numerical analysis. The early solutions were based on the smeared crack model. Later, these were substituted by the discrete crack model. The model adopted within FDEM is actually a combination of smeared and discrete crack approaches [4]. It was designed with the aim of modelling progressive fracture and failure including fragmentation and of creating a large number of rock fragments. For that purpose, the strain softening which appears in the material after reaching the tensile or shear strength is described in terms of displacement. The separation of the surfaces of the adjacent finite elements induces a bonding stress (see Fig. 3a) which is assumed as a function of the size of separation δ (see Fig. 3b).

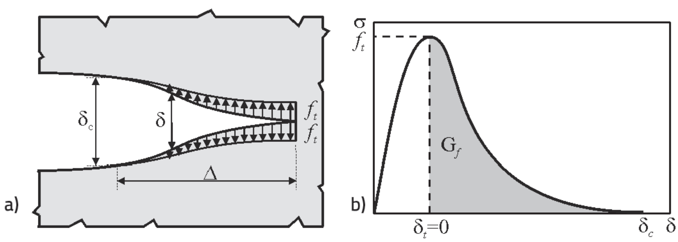

Figure 3. a) Single crack model [4], b) Strain softening defined in terms of displacement [4]

The area under the stress-displacement curve represents the energy release rate G_f $=2\gamma$, where γ is the surface energy, i.e. the energy needed to extend the crack surface by a unit area. Theoretically, there is no separation δ before reaching the tensile strength. In the actual implementation, it is enforced by the penalty method. For the separation $\delta \leq \delta_t$, the bonding stress is given by:

$$\sigma = \left[\frac{2\delta}{\delta_t} - \left(\frac{\delta}{\delta_t} \right)^2 \right] f_t \tag{4}$$

where $\delta_t = 2hf_t/p$ is the normal separation inducing the bonding stress equal to the tensile strength f_t, h is the size of the finite element, and p is the penalty term. Hence, the relative displacement error is independent of the finite element size.

After reaching the tensile strength f_t, the stress decreases with an increase of the normal separation δ, whereas at $\delta = \delta_c$ the bonding stress tends to zero. For the separation $\delta_t < \delta < \delta_c$, the bonding stress is given by:

$$\sigma = z f_t \tag{5}$$

where z is the scaling function representing the softening behaviour of the concrete. According to Hillerborg [1], it is used for approximating the experimental stress-displacement curves for the concrete:

$$z=\left[1-\frac{a+b-1}{a+b}e^{\left(D\frac{a+cb}{(a+b)(1-a-b)}\right)}\right]\left[a(1-D)+b(1-D)^{c}\right] \tag{6}$$

where a=0.63, b=1.8 and c=6.0, while the damage parameter D is determined according to the expression:

$$D=\begin{cases}(\delta-\delta_t)/(\delta_c-\delta_t), & \text{if} \quad \delta_t<\delta<\delta_c; \\ 1, \quad \text{if} \quad \delta>\delta_c\end{cases} \tag{7}$$

The same formulation can be used for other semi-brittle materials using appropriate parameters, as obtained by experimental research.

2.3 Description of the numerical model

The bell tower is made of masonry where bricks are bonded with lime mortar [7, 8, 9]. In order to shorten the calculation time, the calculation was made at the macro level, which means that the discretization of each block and mortar was not performed separately, but the discretization of the structure was made by an irregular finite element network with average properties in terms of modulus of elasticity, tensile strength and shear to expect for this type of structure. For the purposes of numerical analysis, the bell tower is discretized with 1220 triangular finite elements.
The geometry of the model itself is shown in Figure 4 (a), and the finite element grid used in the numerical analyses in Figure 4 (b). Contact elements have been implemented between the finite elements for the purpose of simulating the occurrence and development of cracks in the structure.
The tensile strength of the contact elements taken was selected in the amount of 0.2 MPa, and the shear strength in the amount of 0.8 MPa. The coefficient of friction was adopted in the amount of 0.7. In order to take into account the spatial geometry in the plane analysis, the thickness of the edge finite elements at a width of one meter from the edge was selected in the amount of 5.8 m while the thickness of the middle finite elements was selected in the amount of 2.0 m which is actually twice the width of the wall that simulates a front wall parallel to it. The modulus of elasticity of finite elements was chosen in the amount of 2000 MPa and the Poisson's ratio in the amount of 0.3. The corresponding masses were added and the nodes of the mesh at the elevation of the church cornice were retained, and the transmission acceleration to that elevation was not taken into account.

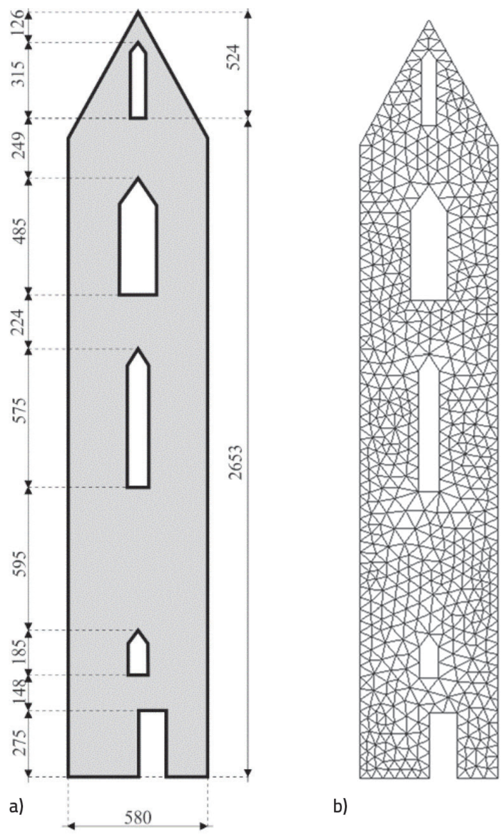

Figure 4. The bell tower: a) model geometry; b) finite element mesh

The Petrovac earthquake accelerometer, often used in the literature, was chosen for the earthquake acceleration diagram. The acceleration diagram is shown in Figure 5, and is proportionally increased ('scaled') to 0.2 g, which corresponds to the maximum acceleration that occurred in the Zagreb earthquake (0.17 g).

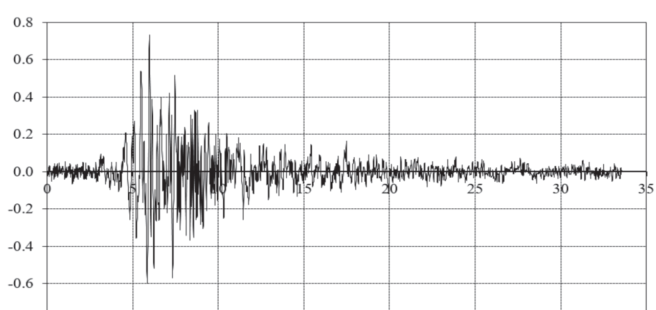

Figure 5. Earthquake acceleration diagram - accelerogram Petrovac

3 Results and conclusion

The cracks that appeared in the numerical model during the action of the given accelerogram (with the largest ordinate 0.2g), and then the collapse of the structure, are shown in Figure 6.

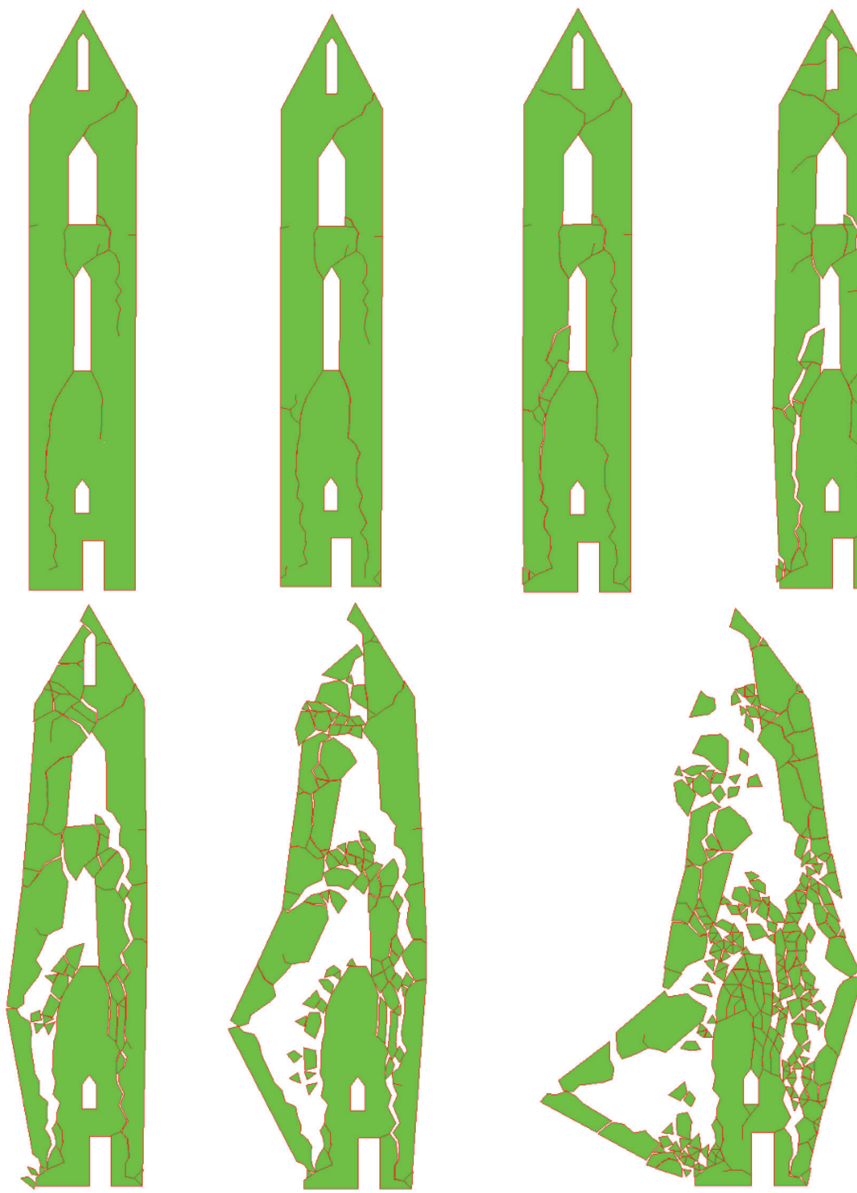

Figure 6. The time history of the cracks on the numerical model and the failure under the accelerogram ($a_{gr.max}$= 0.2g)

Although the accelerogram of the earthquake that occurred in Zagreb was not applied, in the qualitative calculation of the bell tower by the FDEM method, the cracks in masonry were very similar to the cracks that occurred as a result of the earthquake were obtained, as shown in Figure 7.
The numerical model did not have sufficient mechanical resistance for the applied earthquake with a maximum acceleration of 0.2g.
From the coincidence of the positions of the cracks obtained on the numerical model and the cracks on the bell tower, it can be concluded that the bell tower did not have any significant reserves in bearing capacity during the Zagreb earthquake (March 22, 2020). According to a free estimate, a 20-25% higher earthquake intensity or an earthquake with less favorable dynamic properties (more waves with a predominant period of oscillation) would have caused a complete collapse of the bell tower.

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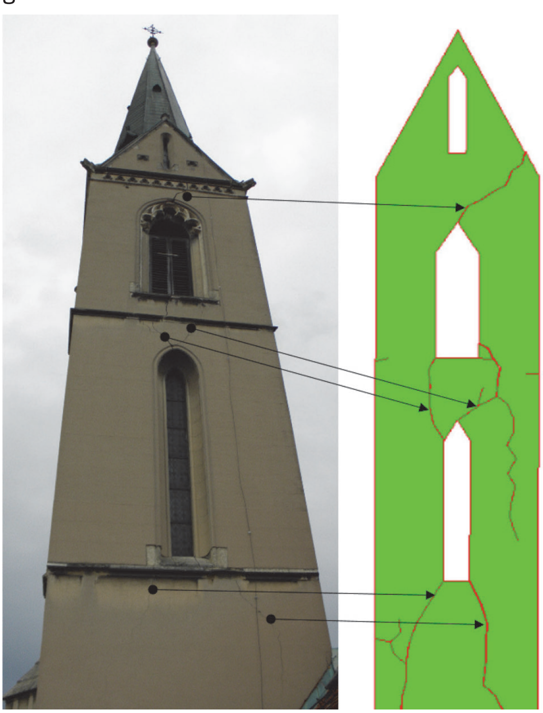

Figure 7. Comparison of cracks on the bell tower and on the numerical model

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