Structures 41 (2022) 684–723

Contents lists available at ScienceDirect

Structures

journal homepage: www.elsevier.com/locate/structures

Review of finite element model updating methods for structural applications

Suzana Ereiz [a,*], Ivan Duvnjak [a], Javier Fernando Jiménez-Alonso [b]

[a] University of Zagreb, Faculty of Civil Engineering, Zagreb 10000, Croatia
[b] Universidad de Seville, Escuela Superior de Ingeniería, Seville 4109, Spain

A R T I C L E I N F O

Keywords:
Finite element model updating (FEMU)
Static testing
Dynamic testing
Structural Maintenance
Finite element modelling
Civil Engineering structures

A B S T R A C T

At the time of designing structures up to date, the density and magnitude of the load have increased, and the requirements for regulation have also become more stringent. To ensure the essential requirements, especially the mechanical resistance and stability, the numerical modelling of the structure is carried out according to the current regulations. Due to various assumptions, idealization, discretization, and parameterizations that are introduced numerical modelling, obtained numerical model may not always reflect the actual structural behavior. It is known that these structures have a hidden resistance that can be determined by combining experimental investigations (static or/and dynamic tests) and finite element model updating methods to minimize the differences between the actual and predicted structural behavior. This paper provides a review of the FEMU process and methods used and summarizes the FEMU approach to help future engineers to select the appropriate method for solving some discussed issues. First, the main terms important for understanding FEMU are introduced. The whole process of model updating is described step by step: selection of updating parameters (design variables), definition of the model updating problem, its solution using different FEMU methods. An overview of the following methods is given: sensitivity-based, maximum likelihood, non-probabilistic, probabilistic, response surface and regularization methods. Each of the method is presented with the corresponding mathematical background, implementation steps, and examples of studies from the literature.

1. Introduction

Numerical models are an effective modern tool for continuous monitoring of structures, damage detection, prediction of service life, and determination of an optimal maintenance strategy. The growing number of new and the advance of current numerical modelling methods have led to the need for numerical models to fulfil stringent requirements related to the accuracy and reliability of model and the results. Therefore, the errors and uncertainties associated with model assumptions that most often lead to inaccuracies and uncertainties must be quantified. Their evaluation is important to determine the degree of reliability and accuracy of the numerical model. This has led to the development of finite element model updating (FEMU) methods that aim to calibrate the numerical model based on the actual behavior of the structure determined as a part of static and/or dynamic testing of structure. In the context of different types of structures, numerical modelling is usually performed using finite element (FE) models [1]. This type of models is used to analyse the internal forces, stresses, displacements, and structural dynamic parameters [2]. While updating the finite element models, there are two possible uncertainties, one related to the predicted FE model and one related to the experimentally obtained data. Uncertainties associated with the FE model include differences between the predicted behaviour (numerical model) and the actual behaviour of the structure. In practise, this error can be reduced but never eliminated. Modelling uncertainties can be generally divided into the uncertainties of the model parameters, the model structure, and the model code [3]. Uncertainty in model parameters is usually due to incorrect assumptions of model parameters such as material properties; section properties, and thickness of shell or plate elements [4]. The uncertainty of the model structure arises from incorrect assumptions about the mechanical properties and physical behaviour of the structure. Such erroneous assumptions arise from different idealization and simplification of the structure, inaccurate assumption of mass distributions, incorrect modelling of mesh connections, boundary conditions, joints [5-7], and so on. Incorrect assumptions of loads, geometric shape, and structural behaviour (nonlinear/linear) can also lead to obvious uncertainty in the model structure [8]. These types of errors can be eliminated by introducing appropriate modelling assumptions. Some differences and unreliability can be minimized by developing a more

* Corresponding author.

https://doi.org/10.1016/j.istruc.2022.05.041
Received 1 April 2022; Received in revised form 10 May 2022; Accepted 10 May 2022
Available online 20 May 2022

Nomenclature

$\widetilde{M}$ experimental data sets
$\boldsymbol{\theta}$ structural model parameters
$\mathbf{z}$ output model
$\mathbf{M}$ model operator
$\mathscr{M}_m$ class of models
P_M model parameter space
P_O response output space
$\boldsymbol{\varepsilon}$ model uncertainty
$\boldsymbol{\mu}$ measurement uncertainty
$\boldsymbol{\theta}_{opt}$ optimal value of structural model parameters
f_i i^{th} natural frequency
ϕ_i n^{th} mode shapes displacement
n_f size of the frequency residual vectors
w_i^f weighting factor of the i^{th} element of the natural frequency residuals
$r_i^f(\theta)$ the i^{th} element of the natural frequency residual vector
n_m size of the mode shape residual vectors
w_i^m weighting factor of the i^{th} element of the mode shape residuals
$r_i^m(\boldsymbol{\theta}) - i^{th}$ element of the mode shape residual vector
FRF Frequency response function
$H_{exp}(f)$ FRFs of experimental model
$H_{num}(f)$ FRFs of FE model
MF Modal flexibility
$MF_{num}(\theta)$ Modal flexibility of FE model to be updated
$MF_{exp}(\theta_{ref})$ Modal flexibility of experimental model
$MF_{num}(\theta_{ini})$ Initial modal flexibility of numerical model
$\|\bullet\|_{fro}$ Frobenius norm
MSE Modal strain energy
$MSE_{num,i}$ modal strain energy for i^{th} mode of the FE model
$MSE_{exp,i}$ modal strain energy for i^{th} mode of the experimental model
ACC acceleration
TH time history
k node number
n_n total number of nodes
n_t time step of measured acceleration
a_{kl}^* measured acceleration at k^{th} node at l^{th} time step
a_{kl} simulated acceleration at k^{th} node at l^{th} time step
IL influence line
t total number of influence line test points
v total number of load step
Z_{Ck} calculated influence line
Z_{Tk} measured influence line
w_s weight factor of the k^{th} test point
ε strain
n_ε size of the strain residual vector
ε_i^{exp} actual measured strain time histories
ε_i^{num} estimated strains time histories
n_δ size of the displacement residual vector
δ displacements
δ_i^{exp} actual measured deflection
δ_i^{num} numerically estimated deflection $\|\bullet\|$- 2 norm
$P(\theta|\mathscr{M}_m)$ probability density function of the design space in the absence of any data
$P\left(\theta|\widetilde{M},\mathscr{M}_m\right)$ posterior probability density function after the data have been observed
$P\left(\widetilde{M}\middle|\theta,\mathscr{M}_m\right)$ likelihood function of data $\widetilde{M}$ in the presence of parameters θ and the model class $\mathscr{M}_m$
$P\left(\widetilde{M}\middle|\mathscr{M}_m\right)$ normalisation function in the presence of the model class $\mathscr{M}_m$
$E\left(f(\boldsymbol{\theta})\,|\widetilde{M}\right)$ posterior expectation of function of the mean value of the updated parameters
$\mu_{\tilde{x}}$ membership function
$\tilde{x}$ fuzzy set
L(x) left reference membership function
R(x) right membership reference function
m average value
p, q constants value in membership function
S_i sensitivity matrix
OA Optimization Algorithm
NNLSS Non-Negative Least Square Solution
GA Genetic Algorithm
NSGA-II Non-Dominated Sorting Genetic Algorithm-II
HGA Hybrid Genetic Algorithm
SGA Simple Genetic Algorithms
CCGA Cooperative Coevolutionary Genetic Algorithm
ELD Encoding by locations and damage factor
SQP Sequential Quadratic Programming
IRR GA Implicit Redundant Representation Genetic Algorithm
SAA Simulated Annealing Algorithm
GAHA Genetic Annealing Hybrid algorithm
ML-GA Multi-Layer Genetic Algorithm
EnKF Ensemble Kalman filter
X_i^n position of swarms
m number of swarm particle
n number of the iteration
s number of variables
x_{ij}^n position of the particle in the iteration n
v_{ij}^n the velocity of the particle in the iteration n
C_1, C_2 learning factors
$rand_1, rand_2$ random numbers between the zero and one
$Pbest_{ij}, Gbest_j$ the best positions achieved by the i-th agent closest to the target since the beginning of the process
HPSO-NT Hybrid Particle Swarm Optimization with Sequential Niche technique
MWFEM Multivariable Wavelet Finite Element Method
PS-NM Hybrid Particle Swarm–Nelder–Mead
MPSO Modified Particle Swarm Optimization
MOPSO Multi-Objective Particle Swarm Optimization
IEPSO Immunity Enhanced Particle Swarm Optimization
PSO-NN Particle Swarm Optimization-based approach to train the Neural Network
PSO-t-IRS Particle Swarm Optimization algorithm and an enhanced instantaneous response surface
SA Simulated annealing
ΔE energy change
P_r Metropolis Hastings acceptance ratio
K_B Boltzmann constant
T temperature in Kelvin
HS Harmony search
HM Harmony memory matrix
MI Maximum improvisation parameter
HMCR Harmony Memory Consideration Rate
PAR Pitch Adjustment Rate
UKF-HS Unscented Kalman Filter- Harmony Search
RS response surface based method
GRSMU Generalized Response Surface Model Updating
MLSM Moving Least-Squares Method
DEA Differential Evolution Algorithm
r number of factors
b number of samples
v number of design variables
MLP Multi layer perceptron

ANN	Artificial neural network	ACO	Art Colony Optimization
t-IRS	instantaneous response surface method for time-dependent reliability analysis	FKH	Fuzzy- Krill Herd
MCMC	Markov Chain Monte Carlo	β	regularization parameters
TMCMC	Transitional Markov Chain Monte Carlo	$\|\{\theta\}\|_2^2$	l_2 regularization term or norm solution
MAP	Maximum a posterior	MPC	Minimum Product Criterion

detailed FE model or so called a high fidelity finite element model [9]. Detailed modelling can minimize the degree of uncertainty in the model and the number of parameters that need to be updated. Most often this type of models and large scale models require performing the parallel computing in order to reduce the time required to perform model updating. Some of the parallel computing methods are bisection [10-13], domain [14] or loop based [15] decomposition, preconditioned conjugate gradient [16], Davidson algorithm [17], graphics processing unit GPU [18], Branch and Bound [19]. To reduce the error between the experimentally obtained structural dynamic properties and numerically obtained one, a sensitivity-based function with multiple variables is used. To find optimal structural parameters that minimize the chosen objective function, an iterative optimization process is performed. The development of a high-fidelity model FE can help to simplify the process of calibrating large structural models, which is contrary to common ideas about it [20]. The main problem on this type of models is the computational time require to perform any type of analysis, but this problem can be overcome using multi-fidelity FEM [21]. On the other hand, the symmetry and regularity of structures and their numerical models often challenge the solution of large-dimensional matrices, the computation of eigenvalues and eigenvectors. Methods such as graph coloration, group theory [22-24] bisection [25] and so on are used to solve this kind of problems. The uncertainty of the model code is related to numerical uncertainties or technical model uncertainties. These uncertainties are mostly the result of software and hardware errors [3].

The experimental methods and their results most used to update the finite element model include static and dynamic structural tests or data and results obtained as a part of structural health monitoring. The errors that are most common in this field include those due to the imperfection of measuring equipment, random measurement noise, signal processing, and, in general, the problem of post-processing the measurement data [26-28]. In order to obtain a numerical model that represents the real structural behaviour as well as possible, its quality must be evaluated [29]. This assessment consists of three steps. In the first step, the assessment of the idealisation and numerical method errors is performed in order to eliminate or reduce these two types of errors. Then, the correlation analysis between the numerical model predictions and the experimental test results is performed to determine the differences and the extent of correlation between the predictions and the test results, and to determine which design parameters of the numerical model affect the output results. In the third step of the assessment, the quality of the numerical model is assessed after updating the selected design parameters.

The definition of FEMU is not uniformly established in the literature. Marwala et al., [30] write in their study that model updating is developed to correct and improve the FE model of the structure according to the actual behavior. Shahbaznia et al., [31] define FEMU as the process of updating the original numerical model of a structure to better reflect the measured response of the actual structure. Schommer et al., [32] defined model updating as an optimization method. In this method, the objective function defined during the FEMU procedure minimizes the deviation between the structural behavior predicted by the numerical model and the actual structural behavior. Mottershead and Friswell [33] define FE model updating as a procedure to update the numerical model to better reproduce the measured response of the actual structures. In another paper [34], the same authors define model updating as the process by which the response of a FE model gradually approximates the response of the real structure by gradually updating the physical parameters. So, the definition is not uniform, but more or less all authors have the common basis of the definition: updating the numerical model based on the experimentally obtained test results to obtain the actual structural behaviour numerically.

General division of FEMU methods divide them into the manual and automated methods. Although this is a very general classification, it is still very important, because often a combination of these methods leads to much better results and they are often used together [35-39]. This combination of automated and manual methods is usually used to bring the initial numerical model as close as possible to the actual behavior of the structure using manual updating, while automated model updating is performed to further reduce these differences and obtain a more reliable estimate of the unknown parameters. In addition, this combination can improve complete process of model updating and speed up computational time [36,37]. Generally, manual methods rely on trial and error in the selection of structural parameters such as geometry, material properties and boundary conditions. They are used when the number of parameters to be updated is small [40]. This method is not able to provide a reasonable physical explanation for the changes in the results. This can lead to inefficient results despite its simplicity [41]. When more parameters are considered, it is recommended to use automated methods. These methods are commonly used to reduce idealization errors [5] and they are introduced with two sub-methods. The first one is a global FEMU and the second one is a local FEMU. The global model update assumes that the uncertainty parameters in the overall model have a single value for each selected element. The local model updating assumes that each mesh element has its own value for the uncertainty parameters [42].

The second classification is a bit more concrete. It divides FEMU methods into non-iterative (direct) and iterative (indirect). Direct model updating methods are the oldest methods used to update numerical models [43,44]. They are used to directly update the structure FEM by changing the structural stiffness matrix and the mass matrix. Without the use of iterative procedures, these methods can reproduce accurate experimental data, which makes them computationally efficient. These methods include the matrix update methods [45], the optimal matrix methods [46], the eigenstructure assignment methods [47], and the Langrage multiplier method [48]. As there are no direct changes in physical parameters of FEM when the model updating is performed under direct methods, the importance of the numerical model decrease, i.e., its ability for simulation decreases [49]. Despite the computational efficiency demonstrated in numerous studies [45,50,51], the use of the direct model updating method has decreased, and it has been replaced by indirect (iterative) methods [50]. The iterative methods are the most commonly used method for performing FEMU of civil engineering structures. They are further divided into deterministic (maximum likelihood) and stochastic (uncertainty quantification) methods.

This paper gives a review of the most used approaches in finite element model updating of civil engineering structures, focusing on the iterative stochastic and deterministic methods and the process of their application. First, the main terms important for understanding the finite element model updating procedure are introduced. Then, the process of selecting the updating parameters is discussed. The mathematical formulation of finite element model updating problem for deterministic and stochastic iterative methods are discussed in the section 4. Sections 5-11 discuss the deterministic and stochastic iterative methods most often used for FEMU. Each of the methods is presented with the appropriate mathematical background and examples of studies in which it has been used to improve simple and complex numerical models and structures.

2. Main terms in FEMU procedure and their relationship

Before starting with the procedure and methods of FEMU, definition and explanation of some main terms is required. Those terms include the model, model class, measured data and model updating. In the following their definition and mathematical description is given.

Experimental datasets $\widetilde{M}$ is a q-component vector that can compress one type of dataset (homogeneous) or multiple types of datasets (heterogeneous). This vector is based on the output quantities such as the natural frequency and mode shapes, while it can also be based on the strains and displacements.

Considering the structural properties as input variables and the results of numerical analysis as the output variables. The model can be generally described as the input–output function between the updating design variables and output results. Input variables considering the structural model parameters $\boldsymbol{\theta}$, while the output response $\mathbf{z}$ is defined as the output (i.e., displacements, strains, natural frequency, mode shapes…) due to the any input vector $\mathbf{x}$. To sum previous, can be generally defined by the equation that connects the input and the output variables in the following form:

$$z = \boldsymbol{M}(\boldsymbol{x}, \boldsymbol{\theta}) \tag{1}$$

where $\mathbf{M}$ is model operator which describes the input–output behaviour. In the finite element model updating procedure it is often working with the output results which are independent of vector $\times$, and the previous relation can be expressed as:

$$z = \boldsymbol{M}(\boldsymbol{\theta}) \tag{2}$$

Model parameter vector $\boldsymbol{\theta}$ represents a class of models $\mathscr{M}_m$ and ranges over a subset P_M. The model in the structural model class can be defined as:

$$\mathscr{M}_m = \{\mathscr{M}_m(\boldsymbol{\theta}) \,|\boldsymbol{\theta} \in P_M\} \tag{3}$$

Each of the associated model in model class maps the model parameter space P_M into the model output response space P_O.

After defining the experimental data sets, model, model class, the model updating can be defined as a process of parameter estimation of specific model class. If considering the vector of model uncertainty ($\boldsymbol{\varepsilon}$) and vector of measurement uncertainty ($\boldsymbol{\mu}$), the vector of measured data sets can be defined as follows:

$$\widetilde{\boldsymbol{M}} = \boldsymbol{M}(\boldsymbol{\theta}) + \boldsymbol{\varepsilon} + \boldsymbol{\mu} \tag{4}$$

For model parameter optimal value $\boldsymbol{\theta}_{opt}$, the outputs of numerical model $\boldsymbol{M}(\boldsymbol{\theta}_{opt})$ represent a model $\mathscr{M}_m(\boldsymbol{\theta}_{opt})$ for the experimentally obtained data sets $\widetilde{\boldsymbol{M}}$.

The next section discussed about the selection of design variables whose values are updated when the finite element model is updated.

3. Selection of the updating parameters and model class

The selection of an appropriate set of parameters of the numerical model, whose values are updated during the model updating is a non-trivial procedure. The selected parameters of the numerical model should represent the unknown structural properties, but their number is also be limited to avoid ill-conditioned problems. However, in addition to adjusting the number of updating parameters, ill-conditioned problems can also be solved by conditioning the structural stiffness matrices using the cut-set basis [52] and preconditioned conjugate gradients methods [53], flexibility matrices using the methods for generating cycle bases of networks [54] and other methods [55].

Regardless of the method used to perform model parameterization, problems arise during updating that lead to non-unique solutions. Parameter estimation is constrained when the amount of measured data is insufficient. This leads to an underdetermined system of equations in deterministic methods or unidentified parameters in stochastic iterative methods [2956]. Regularisation is often used to update the deterministic finite element model, but parameterization is also preferred [57]. Regardless of the simplicity or complexity of finite element models, they often have many parameters, including the material properties and cross section of the element, the connection of model and boundary condition properties, and the model geometry, which can be selected as updating parameters. Model parameterization has a significant impact on reducing errors and simplifying the finite element models. According to Mottershead and Friswell [33] in order to meet the requirements for the accuracy and reliability of the numerical model and the performance of the model updating procedure, the parameterization procedure should meet the following criteria:

- To overcome the ill-conditioned problems, a limited number of parameters should be selected for updating.
- The uncertainties model should be corrected by model parameterization.
- The outputs of the numerical model must be sensitive to selected updating parameters.

Model parameterization that includes sensitivity analysis of the model has the great advantage of providing sensitive parameters and suppressing the problem of inadequacy. This group of parameterization methods includes the subset selection method [58] and the parameter clustering method [59]. In the subset selection method, a reduced number of parameters of the finite element model are selected to be used as updating parameters. The parameters that do not affect the output results are excluded from the model updating process. Originally, this approach was used in regression analysis [60]. Since it is not practical or possible to test all possible subsets of parameters for a large number of parameters, heuristics are used [61]. In these approaches, parameters are selected by an orthogonalization process based on the similarity of their sensitivity vector corresponding to the columns of the sensitivity matrix. The orthogonalization process ensures that each parameter has a different effect on the residual reduction. In addition, there are methods based on the decomposition of the sensitivity matrix [62] and the method that uses global sensitivity analysis for subset selection in model updating [58]. The second sensitivity-based method, the clustering method [59], is based on grouping the parameters of a numerical model with similar sensitivity into a cluster, each of which changes with an update parameter [63]. Selected updating parameters from the same cluster have the same effect on the model updating process. To link similar sensitivities, the unweighted pair group method (UPGMA) is used along with the arithmetic mean [64]. This method allows grouping parameters into binary clusters and then all uncertain parameters are normalized to specific values based on their physical values. To obtain an updated model for further analysis, the updated parameters are multiplied by their initial value. In addition to the previously described parameterization methods based on sensitivity analysis, some other iterative methods are also used for parameterization, such as Bayesian parameterization [65] and particle swarm parameterization [66].

Despite the proposed techniques, the selection of updating parameters depends mainly on the understanding of structural principles, good engineering judgment, and test objectives [29,67]. In order to obtain a physically accurate model, avoid convergence difficulties and ill-conditioned problems, the number of updating parameters must be limited and correspond to the test objective. Ultimately, it should provide an updated analytical model that represents a real structure and its actual behavior [38,68].

In addition to properly defining parametrization and selection of the updating parameters, properly performing the selection of the class of the structural model is also important for the successful and efficient updating procedure. The class of numerical model represents a set of probable input–output models of the modelled system with respect to the various parameterizations of the structure [69]. To perform the model class selection different methods can be used such as the sensitivity-based method [70], the Bayesian approach [71], and the Particle Swarm Optimization [66]. Most often, the model class selection is performed using the Bayesian approach due to the fact that it gives a quantitative expression that can be used to set those simpler models are to be preferred over unnecessarily complicated [65]. According to this method, the model class with the highest probability is selected for further use. It often happens that a complex model class is better than one that has less adjustable uncertain parameters, which is a problem. If the selected model class, which is considered optimal in each class, minimizes the rate of fitting error between the output data and the corresponding predictions, the choice of model will tend to those that have more efficient free parameters. Therefore, in choosing the optimal model class, it is very important to penalize the complicated model, which is a great challenge [71]. This topic is very important in order to select the numerical model which best describes the actual structure without compromising the computational efficiency of the model updating process and the accuracy of the adjustment [72]. This procedure is very important in model updating and the closely related procedure of selecting a model class that most accurately describes the actual behavior of the structure without compromising the complexity of the model, the computational efficiency of the improvement method, and ultimately the results of the model updating. Therefore, this topic will be discussed through the some of the following chapters that deal with the model updating methods that are also used for model class selection.

4. Definition of finite element model updating problem

The finite element model updating problem is generally defined as the difference between the structural behavior predicted by the numerical model and the actual behavior. Depending on the method used, whether iterative stochastic or deterministic, this problem is defined as an optimization or statistical problem. This section gives an overview of the definition of finite element model updating problem using the iterative deterministic or stochastic (probabilistic and non-probabilistic) method.

4.1. Iterative deterministic maximum likelihood method

In practical engineering applications, FEMU is performed using the maximum likelihood method (MLM), which transforms the model updating problem into an optimization problem. As part of the transformation, an objective function is defined in terms of residuals between different types of numerically and experimentally obtained data sets (Tables 1 and 2). These data sets include the structural dynamic properties [73-80], static data sets [81-84] or their combination [32,85-88]. The most often, as a first indicators the structural dynamic properties - natural frequencies and mode shapes are used. These data sets are the best indicators of the actual behaviour of the structure, because when there are changes to the structure, this leads to a change in the structural stiffness (structural flexibility), which in turn leads to a change in the structural dynamic properties. These changes are not large and emphasize. Therefore, it is very important to achieve high accuracy when performing the experimental tests in the field. In addition to the natural frequencies and mode shapes, the formulation of the objective using the frequency response function (FRF) in FEMU is also very popular [75,89-95] and offers some advantages in the application. These advantages are related to the fact that the FRF can adequately reproduce the dynamic properties. Moreover, by using the FRF, the FEMU avoids the error caused by modal fitting and does not require any fitting between the predicted and measured mode shapes [75]. Other widely used forms of the objective function that have also been successfully used in FEMU are the modal flexibility residuals (MF) [77,78,96-98]. Comparing the influence of different possible residuals (frequency, mode shapes, and modal flexibility and their combination), the authors conclude that [77] the objective function that considers all three residuals shows the best performance in model updating. In addition to the

Table 2
Examples of objective function defined using static data sets.

Data sets	Examples of related studies	Example of objective function
IL	Liao et al., [81]	$\sum_{s=1}^{t} w_s \sum_{k=1}^{v} \left(\frac{Z_{Ck} - \eta Z_{Tk}}{Z_{Tk}}\right)^2$
ε	Tchemodanova et al., [82]	$\sum_{i=1}^{n_\varepsilon} \left(1 - \frac{\|\varepsilon_i^{exp} - \varepsilon_i^{num}\|}{\|\varepsilon_i^{exp} - \varepsilon_i^{\overline{num}}\|}\right)$
δ, ε	Kim et al., [83] Sanayei et al., [84]	$\sum_{i=1}^{n_\delta} \left(1 - \frac{\|\delta_i^{exp} - \delta_i^{num}\|}{\|\delta_i^{exp} - \delta_i^{\overline{num}}\|}\right) \sum_{i=1}^{n_\varepsilon} \left(1 - \frac{\|\varepsilon_i^{exp} - \varepsilon_i^{num}\|}{\|\varepsilon_i^{exp} - \varepsilon_i^{\overline{num}}\|}\right)$

Table 1
Examples of objective function defined using dynamic data sets.

Data sets	Examples of related studies	Example of objective function
f_i, ϕ_i	Jiménez-Alonso et al., [110]	SINGLE OBJECTIVE $f(\theta) = \frac{1}{2}\left[\sum_i^{n_f} w_i^f \bullet r_i^f(\boldsymbol{\theta})^2\right]^{1/2} + \frac{1}{2}\left[\sum_i^{n_m} w_i^m \bullet r_i^m(\boldsymbol{\theta})^2\right]^{1/2} \theta \in [\theta_l, \theta_u] \& \sum w_i = \sum w_i^f + \sum w_i^m = 1\, w_i \geq 0$ MULTI OBJECTIVE $min f(\theta) = min(f_1(\boldsymbol{\theta}) \& f_2(\boldsymbol{\theta})) \begin{cases} f_1(\theta) = \frac{1}{2}\left[\sum_i^{n_f} r_i^f(\boldsymbol{\theta})^2\right]^{1/2} \\ f_2(\theta) = \frac{1}{2}\left[\sum_i^{n_m} r_i^m(\boldsymbol{\theta})^2\right]^{1/2} \end{cases}$
FRF	Pu et al., [91]	$\boldsymbol{H}_{exp}(f) - \boldsymbol{H}_{num}(f)$
MF	Cui et al., [97]	$\frac{\|\boldsymbol{MF}_{num}(\boldsymbol{\theta}) - \boldsymbol{MF}_{exp}(\boldsymbol{\theta}_{ref})\|_{fro}^2}{\|\boldsymbol{MF}_{num}(\boldsymbol{\theta}_{ini}) - \boldsymbol{MF}_{exp}(\boldsymbol{\theta}_{ref})\|_{fro}^2}$
MSE	Jaishi and Ren [79]	$\sum_{i=1}^{n_m} \left(\frac{MSE_{num,i}}{MSE_{exp,i}} - 1\right)^2$
ACC and TH	Feng and Feng [104]	$\sum_{k=1}^{n_n} \left(\sum_{l=1}^{n_t} [a_{kl}^* - a_{kl}]^2\right)$

previously mentioned dynamic properties and their derivatives, the objective function can also be defined using the modal strain energy (MSE) [79,80,99-101]. Measured acceleration is most commonly used for damage detection and estimation of remaining capacity in combination with some of the FEMU methods for structures subjected to traffic-induced vibrations [102-104]. In addition to the use of structural dynamic properties, which are more suitable for modelling complex structures, displacements and strains obtained from in-situ static tests have also been successfully used for FEMU [81-84] performing the FEMU. These types of data sets are also combined with the structural dynamic properties to perform model updating [6,32,36,85,88,105-109].

Since different types of data sets with different nature can be used to perform the model updating, a problem how to weigh the influence of each residuals arise. There are two possible approaches: single (5) and multi (6) objective approach.

$$\min\ f\ (\theta) = \min\ \frac{1}{2}\sum_{i=1}^{n} w_i r_i(\theta)^2 = \min\left(\frac{1}{2}\sum_{i=1}^{n_f} w_i^f r_i^f(\theta)^2 + \frac{1}{2}\sum_{i=1}^{n_m} w_i^m r_i^m(\theta)^2\right) \tag{5}$$

$$\min\ (f_1(\theta),\ f_2(\theta)\,) = \min\ \left(\frac{1}{2}\sum_{i=1}^{n_f} r_i^f(\theta)^2,\ \ \frac{1}{2}\sum_{i=1}^{n_m} r_i^m(\theta)^2\right) \tag{6}$$

In a single objective approach (5), the objective function is defined by the weighted differences (referred to as residuals) between the numerical and experimental properties under consideration. To account for the relative contributions and uncertainties associated with an experimental estimate of a dynamic or static structural parameter to the objective function, the residuals must be weighted in this approach. The weighting of the residuals is very important to obtain more accurate FEMU-a results. When the natural frequencies are taken into account, their values can be determined experimentally very easily and with high accuracy. Therefore, weighting factors with a high value are assigned to them. On the other hand, compared to the natural frequencies, the mode shapes are less sensitive to changes in structural stiffness and have about 10 times greater influence due to noise [51]. In order to achieve a possible correlation between the experimentally and numerically obtained data sets, the weighting factors of the mode shapes must be analysed to obtain their optimal values [111]. Since the optimal value of the weighting factors is not known in advance, they can be obtained by the trial-and-error method [108] or by statistical criteria [33]. Usually, it is assumed that the optimal value is between 0 and 1 [112]. The use of these values ensures the best correlation between experimentally and numerically determined mode shapes. In another work, it was assumed that the optimal value of the weighting factors is different [36]. In damage detection based on FEMU, it is also difficult to define an objective function and choose appropriate weighting factors for mode shapes or natural frequencies when structural dynamic parameters are involved, since it is not known which of them are important for a particular damage detection problem [113]. On the other hand, multi-objective approach (6) uses different objective functions with respect to the different residuals [114]. The general aim of this approach is to find the optimal solution in the Pareto optimal front [115]. To determine the best solution, a reasonable criterion must be defined. In a FEMU problem defined with two sub-objective functions (bi-criterion problems), an additional constraint is applied in most cases, mainly based on the decision-making strategy [116]. This approach tries to find good compromises or "trade-offs" between conflicting objective functions in an optimal manner. Moreover, the most commonly used criteria consider the edge knee point [117]. This criterion is based on finding a solution where a small improvement in one objective would lead to a large deterioration in at least one other objective [117].

Comparing the single and multi-objective approach their advantages and disadvantages can be pointed out. The single-objective approach depends on weighting factors given by subjective preferences, experience, or engineering judgement [118]. In addition, multiple optimization runs may be required to validate the potential models. All alternative updated models are searched in one step and the best model is selected in a single optimization run using a decision strategy, reducing the time required to find the best FE model. To compare the effectiveness of single and multi-objective functions, authors usually perform FEMU using both approaches. To overcome the disadvantage of the computational cost and the unique dependence between the updated model and the objective function considered, Jiménez-Alonso et al., [110] performed a study on a laboratory model of footbridge using both a single and a multi-objective function. Based on the research performed, they concluded that the multi-objective approach is the best option for the FEMU, since it allows a large search space, reduces the computational time, and provides a better balance of the influence of two sets of considered residuals based on the natural frequencies and mode shapes. Naranjo-Pérez et al., [119] validated the performance of the new hybrid algorithm by comparing it with three different computational intelligence algorithms performed for the same real structure as in [110]. The comparison was based on the speed of convergence and accuracy of matching, using both single and multi-objective functions. They also concluded that the multi-objective approach is better than the single-objective approach. Jin et al., [118] performed the comparison between the single and multiobjective approaches and concluded that all the updated models of the single objective approach are behind the optimal Pareto front (far from the origin). On the other hand, it is also found that the weighting factors should balance the sub-objective functions, but in some cases deviate from this expectation. In addition, the updated parameters of the multi-objective approach appeared to contain physical significance with fewer objective function values, while the single-objective approach resulted in about 50% of the updated parameters being close to constraints. Based on the literature review conducted to define the FEM problem as an optimization problem, it can be concluded that the multi-objective method shows better performance in solving the updating problem.

4.2. Iterative stochastic methods

Using the iterative stochastic method to update the finite element model, the FEMU problem is considered as a statistical problem focusing on the quantification of uncertainty. This quantification can be divided into two categories: probabilistic and non-probabilistic. The first category represents the classical approach to modelling uncertainty, which is based on probability theory and in which uncertainty is modelled using the probability density function (PDF). The probability distribution can be defined in different forms: Bernoulli distribution [120], uniform distribution [63], binomial distribution [121], normal distribution [122], Poisson distribution [123] or exponential distribution [124]. In civil engineering applications and finite element model updating, the uniform distribution and the normal distribution are most commonly used [20]. Each set of parameters is assigned a priori probability density function. The assigned functions incorporate prior knowledge or information about the value of the structural parameters and are subject to bias due to the quality and uncertainty of the information. The quantified uncertainty of the prior PDF is updated using Bayes' theorem, which is mathematically represented as follows (Eq (7)):

$$P\left(\theta|\widetilde{M},\mathscr{M}_m\right) = \frac{P\left(\widetilde{M}\middle|\theta,\mathscr{M}_m\right)P(\theta|\mathscr{M}_m)}{P\left(\widetilde{M}\middle|\mathscr{M}_m\right)} \tag{7}$$

where $P(\theta|\mathscr{M}_m)$ is the probability density function of the design space in the absence of any data (prior distribution function) when the model class $\mathscr{M}_m$ is known and the data $\widetilde{M}$ is absent; $P\left(\theta|\widetilde{M},\mathscr{M}_m\right)$ is the posterior probability density function after the data are observed (in the presence of data $\widetilde{\mathrm{M}}$ and the model class $\mathscr{M}_m$; $\mathrm{P}\left(\widetilde{M}\middle|\theta,\mathscr{M}_m\right)$ is the likelihood function of data $\widetilde{M}$ in the presence of parameters θ and model class $\mathscr{M}_m$; $P\left(\widetilde{M}\middle|\mathscr{M}_m\right)$ is the normalisation function in the presence of model class $\mathscr{M}_m$. In the situation where experimentally obtained data sets are constant, the marginal distribution of the data $\widetilde{M}$ does not depend on the model parameters θ and the previous equation (Eq.7) can be rewritten as follows (Eq (8)):

$$\mathrm{P}\left(\theta|\widetilde{\mathrm{M}}\right)\propto\mathrm{P}\left(\widetilde{\mathrm{M}}\middle|\theta\right)\mathrm{P}(\theta) \tag{8}$$

To obtain the mean value of the updated parameters the following equation (Eq (9)) is used:

$$\mathrm{E}\left(\mathrm{f}(\theta)\,|\widetilde{\mathrm{M}}\right)=\frac{\int \mathrm{f}(\theta)\mathrm{P}\left(\widetilde{\mathrm{M}}\middle|\theta\right)\mathrm{P}(\theta)}{\mathrm{P}(\theta)} \tag{9}$$

where $\mathrm{E}\left(f(\boldsymbol{\theta})\,|\widetilde{M}\right)$ is the posterior expectation of the function of the mean of the updated parameters $f(\boldsymbol{\theta})=\boldsymbol{\theta}$, which depends on the posterior PDF. For complex systems, which are usually very large, this integral may not be available. In general, several methods are used to solve this integral. They can be generally divided into numerical evaluation, analytical approximations, and sampling methods. When the posterior distribution function $\mathrm{P}\left(\theta|\widetilde{M}\right)$ is very large, numerical evaluation may not be accurate [125]. On the other hand, analytical approaches are suitable and converge in local minima [126]. They considers maximum likelihood [127], maximum a posterior MAP [126] and Laplace's method [128]. Third, sampling methods are most popular to solve the complex posterior distribution. This method considers method such as Markov Chain Monte Carlo method [128].

A variety of methods have been developed to obtain a priori probability density function. One of the most popular approaches is the conjugate prior [129]. In this approach, the prior PDF is chosen based on the likelihood function such that the posterior PDF is assigned to the same family distribution [130]. The main advantage of this approach is that it greatly facilitates the accurate calculation of the full posterior PDF. The conjugate prior approach is still relevant because it provides better insight into exactly how the data change or update the prior PDF. Other approaches [131,132] assume that the prior PDF should add as little as possible to the available prior information. In addition, some more formal methods for avoiding the addition of information by the prior PDF have been developed, mainly based on information theoretic criteria [133-135]. The most commonly used prior PDF selection approach is based on maximum entropy [135]. This approach is based on expressing the probability distribution as the best representative of the current knowledge about a parameter that provides the largest information entropy. Thus, the method itself leads to a uniform prior PDF when it is known that the parameters in the final range of values are non-zero within a certain interval. If the prior available information contains a mean and a finite variance, the maximum entropy leads to a normal distribution.

A likelihood function is a function describing the probability of the observed data parameterized by θ. It can be determined by applying the law of total likelihood and according to the equations with the probabilistic model of measurement error and modelling error. The likelihood function can be calculated as the convolution of the PDFs of measurement error and modelling error. If the data sets on the individual errors are not available, the likelihood function can be determined using the probabilistic models of the total prediction error, parameterized by θ. Since there is often very little information available about the error properties, the impact of the likelihood function on the process of model updating using the Bayesian method is very large. Therefore, the definition of this function has attracted much attention in recent decades. In terms of the likelihood model, a realistic estimate can be made. The probability model represents a prediction error. It is usually assumed that the probabilistic prediction error model is known and fixed, so that the set of parameters reduces to $\boldsymbol{\theta}=\{\theta_M\}\in\mathbb{R}^{N_M}$. To represent the prediction error in structural dynamics application, an uncorrelated Gaussian model with zero mean is usually used, which is important for better understanding of the expression and calculation. In this approach, entropy is maximised in terms of prediction errors rather than model parameters. This model is not applicable in situations where errors have correlation [136] or in situations with system components [137]. To prevent the influence of incorrect and inappropriate assumptions on the results of Bayesian model updating, identification methods are used [138]. One of them incorporates error parameters (variance of the uncorrelated Gaussian model with zero mean or correlation parameters in terms of correlation length) into the Bayesian scheme and a reasonable assumption about the model class [71]. The second method uses Bayesian model class selection to determine the class that is most appropriate based on the available information. This category is divided into two subcategories. The first subcategory is used to distinguish several alternative classes of model predictions to eliminate or at least reduce model errors. The second subcategory applies model class selection to determine the most appropriate probabilistic model class that represents the prediction error based on the available information. Once the prior probability density function and the likelihood function have been determined, the following equation allows the PDFs of the model parameters to be updated based on the results of the experimental investigation.

When dealing with real constructions and real systems, the computation of joint and marginal PDFs involves a large number of parameters. This leads to high-dimensional integrals for which approximate measures or sampling methods, such as the Markov Chain Monte Carlo method, are used to solve. If a conjugate prior is used, the posterior probability density function can be determined numerically. If the number of parameters is limited, the posterior PDF can be determined analytically. After the posterior PDF is calculated, estimated, or approximated, it can provide information on how much the uncertainty of the parameters decreases relative to the observed data and the available prior information. The posterior probability density function can be approximated in a different way, including the Gaussian distribution, asymptotic approximations, or sampling techniques. If both the prior PDF and the likelihood function are Gaussian, the posterior PDF will also have a Gaussian distribution [127]. Asymptotic approximations are used when a large amount of data is available. Here, the posterior PDF is approximated by a Gaussian PDF at the point of maximum posterior or MAP and characterized by a covariance matrix [139]. The most popular Markov Chain Monte Carlo method is used to sample the posterior PDF and improve the convergence speed.

On the other hand, the non-probabilistic approach uses a random matrix theory [140] to construct the prediction operator [141]. Most of the proposed non-probabilistic approaches are mainly based on interval analysis [142], where the uncertainties of the variables are represented by a certain range of values. The defined ranges of values are transferred to the outputs of interest (interval analysis). One of the most popular extensions of interval analysis is fuzzy set theory [143]. Fuzzy set theory offers the possibility to model uncertainty if, in addition to the interval bounds, the desired reliability values or confidence levels are available with respect to uncertain quantities. In this approach, the classical binary concept of a set, according to which an element either belongs or does not belong to a set, is replaced by a more intuitive description of the set, where the membership is determined stepwise by the so-called membership function $\left(\mu_{\widetilde{x}}\right)$. The strength of the application of fuzzy set theory can be seen in the stepwise description of membership, which can be interpreted differently depending on the application. Fuzzy finite element model updating (FFEMU) considers the fuzziness in a design variable in the finite element framework [144,145]. It uses the fuzzy theory based on classical set theory [146]. The difference between these two theories is that classical set theory distinguishes between members and non-members. Fuzzy theory, on the other hand, introduces the degree of membership. The degree of membership weights the possibility of members belonging to the set and is described by the membership function. The membership function is constructed based on some probability measures, e.g., estimated means and standard deviations or confidence intervals, or even based on a complete (estimated) probability density function. In either case, it must be clear that the resulting membership value cannot be interpreted as probabilities. The strength of the fuzzy method lies in the stepwise description of the membership [3]. It can be described and interpreted differently depending on the application [3]. The fuzzy set in which the membership of a particular element × is observed can be defined as follows in Eq (10):

$$\tilde{x} = \left\{ \left(x, \mu_{\tilde{x}}(x) \right) \middle| \left(x \in X, \mu_{\tilde{x}}(x) \in [0, 1] \right) \right\} \tag{10}$$

where $\tilde{x}$ is the fuzzy set, $\mu_{\tilde{x}}$ is the membership function defined for all elements × in the domain X. If the membership function is equal to 1 $(\mu_{\tilde{x}}(x) = 1)$, it means that × is a member of the set $\tilde{x}$. On the other hand, if the membership function is zero $(\mu_{\tilde{x}}(x) = 0)$, it means that the element × is not a member of the set $\tilde{x}$. If the membership function takes values between 0 and 1, $(0 < \mu_{\tilde{x}}(x) < 1)$ it means that the element × is a member of the set $\tilde{x}$ with a certain degree of membership. The membership function defined using the left and right reference functions, the left–right membership function, can be defined as follows in equation Eq (11).

$$\mu_{\tilde{x}}(x) = \begin{cases} L\left(\dfrac{m-x}{p}\right), (x \le m, p > 0) \\ R\left(\dfrac{x-m}{q}\right), (x > m, q > 0) \end{cases} \tag{11}$$

where L(x) is the left reference function; R(x) is the right reference function; m is the average value; p and q are a constant value. The most common forms in which a membership function can be defined are trapezoidal [147], triangular [145], singleton [148], Gaussian [149] and piecewise linear [150] shapes (Fig. 1).

In the analysis of fuzzy FEMU problems, the fuzzy membership function is divided into different sublevels, the most commonly used being the α-sublevel technique [145]. In the α-sublevel technique, a membership function is divided into a set of sublevels. For each sublevel, the lower and upper bounds are defined, and the fuzzy uncertainty propagation is defined as the application of the series to the interval analysis at multiple α-sublevels. An important problem associated with standard fuzzy set theory is the inability to account for the dependence or interaction between fuzzy input variables and outputs. This means that fuzzy modelling always specifies the maximum range of output variables at each α-sublevel because it is implicitly assumed that any combination of values of the input variables is equally probable. Fuzzy uncertainty propagation is performed using two methods: the interval-based and the global optimization approaches [151]. The first, interval-based approach is based on standard interval arithmetic and treats the fuzzy variables as an interval for each α-sublevel. The second, global optimization approach is based on two steps. In the first step, the minimum value of the output vector is determined, after which its maximum value is determined in the second step. By combining the obtained results for all defined α-sublevels, the membership function for the output variables is obtained. Thus, the uncertainties associated with the modal parameters can be defined as membership functions, while the model updating procedure is defined as an optimization problem. In this optimization problem, the objective function is defined as the difference between the upper and lower bounds of the experimentally and numerically obtained data sets. The defined objective function must be minimised to obtain interval values for the modal parameters. The obtained modal parameter values are combined to obtain the final fuzzy parameters.

In the following section follows an overview of the most commonly used finite element model updating methods: sensitivity-based FEMU, FEMU using a nature-inspired computational algorithm, Response Surface FEMU, FEMU under the non-probabilistic and probabilistic approach, and FEMU using the regularization method. All of these methods are presented with the appropriate mathematical background and examples of studies and research conducted in the field of FEMU and their application to damage detection, reliability analysis, model class selection, optimal sensor placement, and so on.

5. Sensitivity based model updating

The main idea of the sensitivity-based method is based on the linearization of the generally non-linear relationship between the measurable outputs and the structural model parameters that require correction. It is developed from a Taylor series expansion truncated after

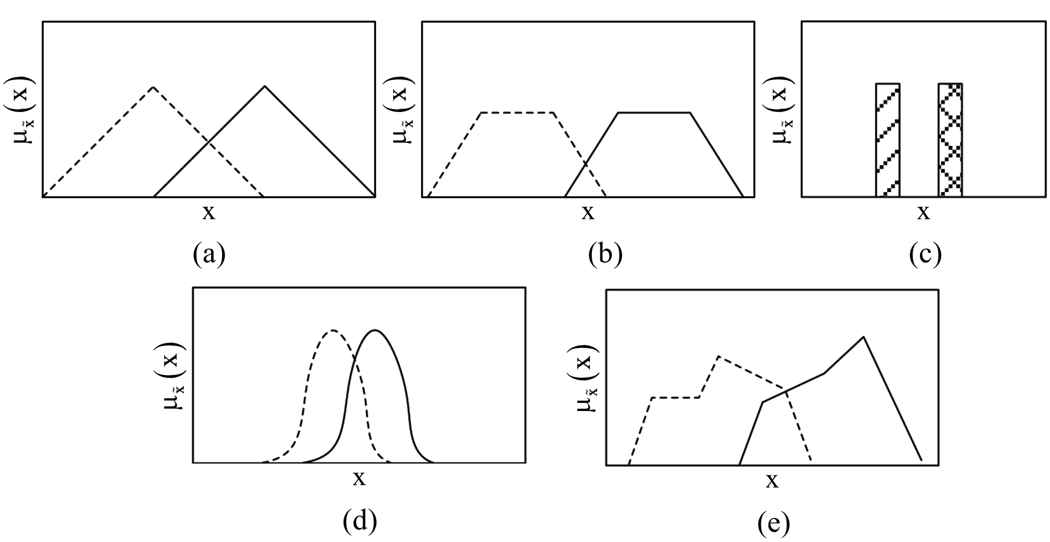

Fig. 1. Graphical representation of a) triangular b) trapezoidal c) singleton d) Gaussian e) piecewise linear membership functions.

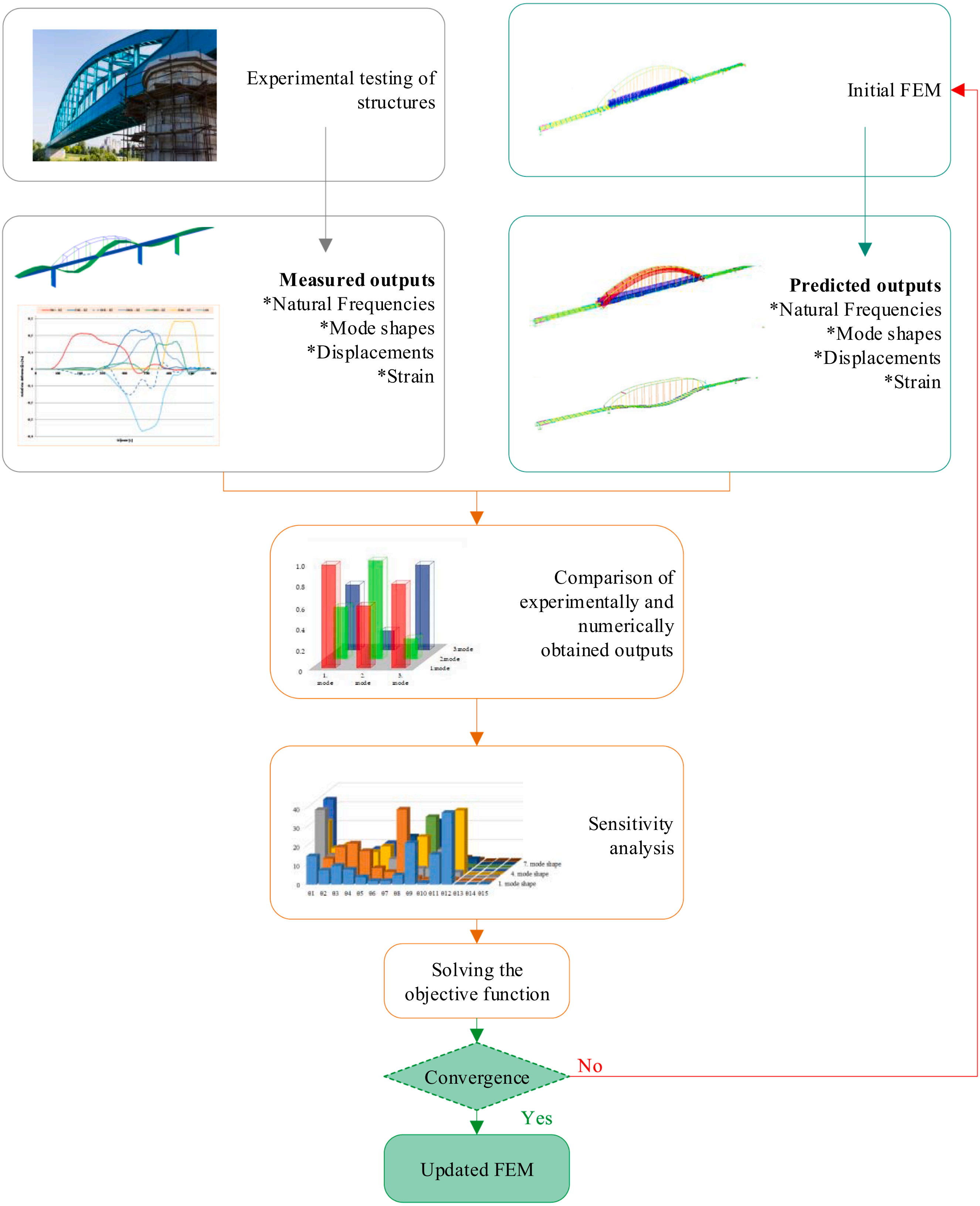

Fig. 2. Sensitivity based model updating method flowchart.

the linear term (Eq (12)):

$$\varepsilon_m = \widetilde{M} - M(\theta) \approx r_i - S_i(\theta - \theta_i) \tag{12}$$

The residual r_i is defined at the i^{th} iteration as (Eq (13)):

$$r_i = \widetilde{M} - M(\theta) \tag{13}$$

so that the linearization is carried out at $\theta = \theta_i$. Experimentally obtained and predicted outputs are denoted by $\widetilde{M}$ and $M_i(\theta) = M(\theta_i)$. The sensitivity matrix S_i is given by Eq (14):

$$S_i = \left[\frac{\partial M_j}{\partial \theta_k}\right]_{\theta=\theta_i} \tag{14}$$

where $j = 1, 2, \cdots, q$ denotes the output data points and $k = 1, 2, \cdots, p$ is parameter index. Sensitivity matrix, S_i, is calculated at the current value of the complete vector of the parameters $\theta = \theta_i$. The error, ε_z, is assumed to be small for parameters θ in the vicinity of θ_i. At each iteration (14) is solved for:

$$\Delta\theta_i = \theta - \theta_i \tag{15}$$

and the model is than updated to give:

$$\theta_{i+1} = \theta_i + \Delta\theta_i \tag{16}$$

This procedure continues until consecutive estimates θ_i and θ_{i+1} are successfully converged. The flowchart of the sensitivity based finite element model updating method is shown on following Fig. 2.

He et al., [5] performed a sensitivity-based FEMU of a skewed highway bridge using monitoring data from wireless measurements to obtain a more accurate numerical model. They formed the FEMU process into a numerical optimization problem. Based on the model updating performed, the authors concluded that the idealization and simplification of the numerical model can be minimized if only model updating is performed instead of parametric updating. The results also show that the superelevation and the cross slope can have a great influence on the dynamic characteristics of the skew bridge. Jiang et al., [152] compared two FEMU methods using frequency response function data by updating the truss model and cantilever. The first method is based on sensitivity analysis, while the second method is based on defining the modelling errors as linear combinations of individual element matrices. These individual matrices can be used for both model updating and fault localization. The comparison showed that the sensitivity method is effective only when the test data is extremely incomplete. Zhu et al., [153] performed sensitivity-based model update of a cable-stayed bridge by using influence line analysis to update the multiscale model with the measured modal frequencies. Ren and Chen [154] compared the response surface-based finite element model updating with the sensitivity-based method tested on a simply supported beam and a full-size continuous precast box girder bridge under service vibration conditions. Based on the comparison, the authors concluded that the response surface based method was more efficient and converged faster than the traditional sensitivity based method. Park et al., [155] proposed an update of the finite element model for a cable-stayed bridge based on a two-step procedure. In the first step, the manual model update is performed based on the initial design information and construction and maintenance history. In the second step, the base model is calibrated using a sensitivity-based optimization. To simultaneously fit the FE mass and stiffness matrices using incomplete, noisy modal data, Rezaiee-Pajand et al., [99] proposed an innovative sensitivity-based FEMU strategy that combines modal kinetic energy (MKE) and modal strain energy (MSE). Using model parameterization and first-order Taylor series, the sensitivity-based FEMU strategy was developed into an inverse problem solved by regularization methods. Mosavi et al., [20] performed an update of a high-fidelity finite element model of highway bridges using a multivariable sensitivity-based approach to match the structural behavior predicted by the numerical model to the actual behavior. Razavi and Hadidi [78] investigated the robustness of sensitivity-based model updating for damage detection of structures with large spaced structures using vibration measurements. Based on their study, they concluded that this method is efficient in damage detection in large space structures and that the process of damage detection is independent of the size and nature of structures. Durmazgezer et al., [73] performed the sensitivity-based FEMU on a

Table 3
Review of the using sensitivity based method for FEMU and damage detection.

Reported application	Examples of related study	Type of Sensitivity based method	Structure
FEMU	He et al., [5]	Sensitivity-based FEMU	Skewed highway bridge
	Jiang et al., [152]		Truss model and the cantilever beam
	Zhu et al., [153]		Cable-stayed bridge
	Ren and Chen [154]		Full size precast continuous box girder bridge under service vibration conditions
	Park et al., [155]	Two step model updating	Cable stayed bridge
	Rezaiee-Pajand et al., [99]	Sensitivity based strategy and regularized solution methods	Two story concrete frame and two-span continuous steel truss
	Mosavi et al., [20]	Multi variable sensitivity based high fidelity FEMU	Highway bridge
Damage detection	Razavi and Hadidi [78]	Sensitivity-based FEMU	Large-spaced structures
	Durmazgezer et al., [73]		Half-scale reinforced concrete portal bare frame
	Li et al., [156]		Seven storey plane frame structure
	Venanzi et al., [157]	FEMU through modal sensitivity analysis	Historic masonry tower
	Jaishi and Ren [159]	Sensitivity based FEMU in combination with OA	Reinforced concrete beam
	Entezami et al., [160]	Sensitivity base model updating in combination with GA	Planar truss
	Blachowski [161]	Sensitivity based strategy and NNLSS	3D truss girder, upper deck arch bridge
	Shahbaznia et al., [31]	Sensitivity base MU using Tikhonov regularization	Railway bridge

half-scale reinforced concrete portal frame for damage detection using measured dynamic response data. Li et al., [156] performed damage detection using sensitivity-based model updating, but on a seven-story plane frame structure and using acceleration measurement. The damage detection performed is based on the transmission of the autospectral power density, which formulates the relationships between two sets of autospectral density functions of the output responses. Venanzi et al., [157] proposed a model updating of a historic masonry tower using a modal sensitivity analysis [158] and a linear single-step method for locating earthquake-induced damage using ambient vibration tests and long-term SHM monitoring data. The proposed improvement of the method considers not only the natural frequencies for damage localization but also the mode shapes. In this way, the authors increased the number of DOF and investigated the robustness of the solution and avoided unphysical results. Jaishi and Ren [159] carried out the sensitivity-based model updating of reinforced concrete beam for damage detection using the dynamic properties determined by laboratory experiments. They formulated the objective function consisting of the residuals of modal flexibility and its gradient. By minimizing the objective function using the sensitivity-based model updating and optimization algorithm, the author provided the damage detection capability. Entezami et al., [160] proposed a sensitivity-based finite element model updating method for damage detection using incomplete noisy modal data. In order to adapt the initial sensitivity formulation of modal strain energy (MSE) to the damage detection problem, an improvement of MSE sensitivity functions based on the Langrage optimization problem was performed. To overcome the misplaced damage equation problem caused by sparsity, ill- conditioning, and rectangularity of the sensitivity matrix, a regularized least square minimal residual was proposed. The accuracy of the proposed approach was verified on designed trusses. Blachowski [161] proposed a three-step method for damage detection in spatial truss structures. In the first step, the sensitivity of the modal characteristics is calculated. In the second step, the sensitivity matrix of natural frequency is used to determine hard-to-identify structural parameters, while the sensitivity matrix of mode shape is used to select damage-sensitive sensor locations. In the third step, two algorithms based on norm regularization or non-negative least squares (NNLSS) are used to efficiently identify the applied damage scenarios. In the studies presented, the non-negative least squares solution showed better performance. Shahbaznia et al., [31] proposed a sensitivity-based model updating in the time domain for railway bridge structures under an unknown moving load, considering the interaction between the bridge and the vehicle. By using sensitivity analysis and Tikhonov regularization, the computational cost is drastically reduced.

Recent research and examples of studies dealing with the implementation of model updating using the sensitivity-based method and the application are summarized in Table 3.

Previous research (Table 3.) showed the sensitivity-based methods allow a large number of the updating parameters and measured outputs and most often require a high computational effort, but this problem can be solved by combining the sensitivity based with some other methods such as regularization. Moreover, the sensitivity equation (Eq. (12)) is generally a nonlinear equation linking the input parameters of the

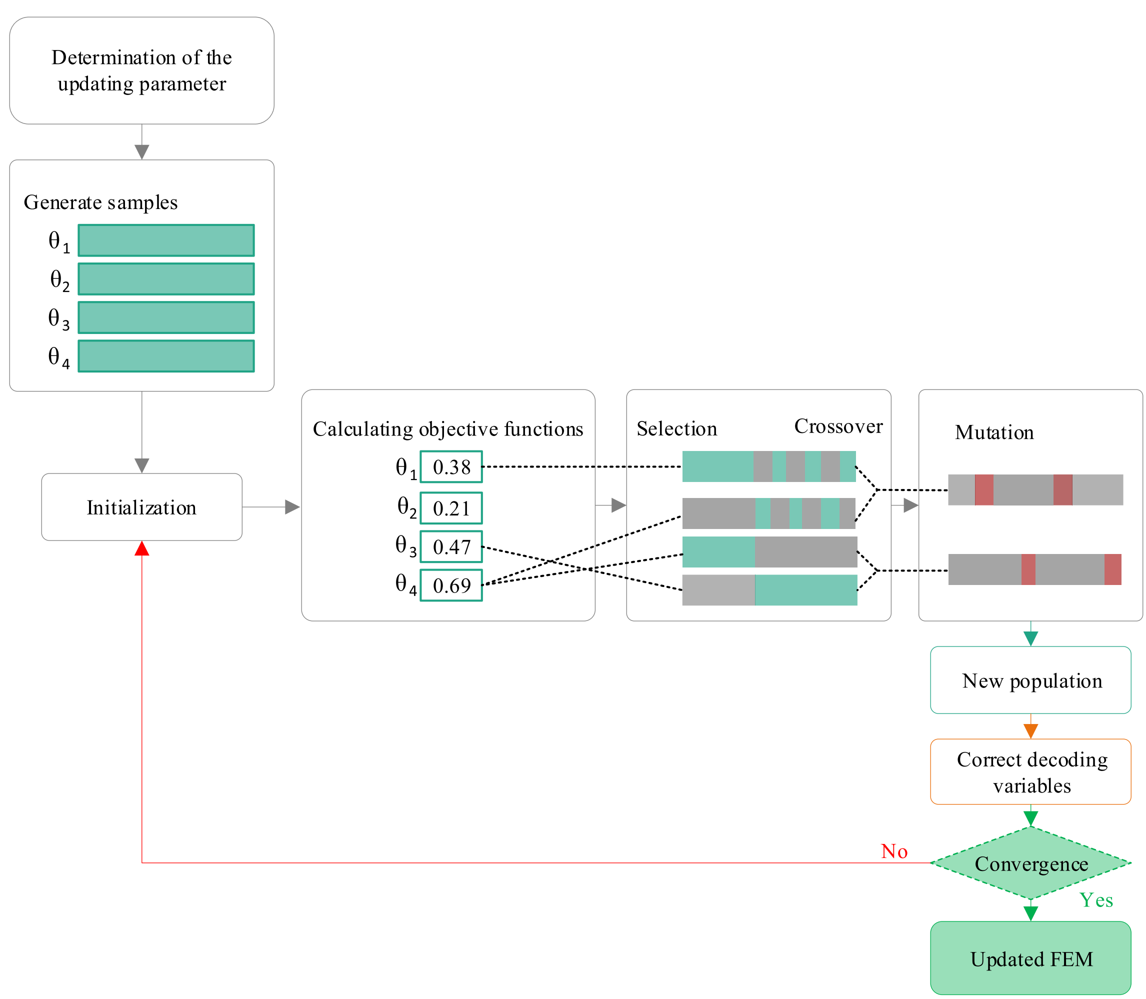

Fig. 3. Flowchart of Genetic Algorithm method.

numerical model and its output, so an iterative procedure must be performed. Consequently, there may be a convergence problem in determining the updating parameters values. In the following section, the iterative optimization procedure, and the application of the nature-inspired computational algorithms, especially the genetic algorithm and particle swarm optimization, in updating the finite element model are discussed.

6. Nature inspired computational optimization algorithms

Iterative finite element model updating techniques are based on the use of changes in the physical structural parameters to perform the model updating and produce the models that are physically realistic [27]. As mentioned earlier, these methods are based on solving an optimization problem for which computational optimization algorithms are usually used to find the global optimum. In this chapter, the genetic algorithm and particle swarm optimization are discussed in detail. For more information and application of the other intelligent algorithms, the reader is referred to the following [9].

6.1. Genetic algorithm

Genetic algorithm represents a stochastic global searching technique based on the global evolution process and Darwin natural selection [164] that was first presented and simulated for FEMU application by Levin and Lieven [165]. It operates to find a solution of an optimization problem for population of possible solutions. In most of the optimization problems, GA works with the initial population size. This population covers a good representation of the updating solution space, and its size should depend on the nature of the optimization problem. The nature of the optimization problem is determined by a set of structural model parameters, which may be updating parameters. From the initial population through the crossover, mutation and selection phases a better generation are iteratively formed. Through phase of crossover, new individuals are generated by combining the random parent chromosomes. Mutation phase is used as an auxiliary method that creates new individuals to avoid the information that is lost in selection and crossover phases. In this phase, the parts of the parent chromosomes are selected and inverted. In this way, new information is introduced into the population [127], diversity of population is maintained, and the local search ability of GAs is improved [166]. To grow a new population from each generation, selection is performed using the fitness-based method. The population that is fitter has a higher probability of being selected. Selection can be done by ranking the fitness of each solution and selecting the best solution or by ranking a randomly selected sample of the population (computational efficiency) using methods such as uniformly order selection [127], stochastic tournament selection [166], and roulette wheel selection [167]. The previous phases are repeated until the stopping criteria are reached (Fig. 3). Commonly used stopping criteria include the maximum number of generations/iteration [168-171], the minimum fitness value [172,173] or any combination of these criteria.

Most early studies focus on the using the GA in performing model updating searching form the properties of structural materials [35], investigating the effect on temperature on the modal frequency [4], for help in understanding the current state of structural restauration [174], etc. Ye and Chen [4] performed the FEMU of the TV tower based on GA to investigate the effects of the different effects of temperature on the modal frequency under two situations. In the first case, they proposed that temperature only affects the elastic modulus, while in the other case, they proposed that it only affects the geometric stiffness of structures. The first situation was considered more important because of the significant frequency change. In the second case, the frequency change was very small, and it was suggested to be ignored. Genetic Algorithm can be very useful for structural identification of the historical buildings and structural health monitoring performed on it. Bianconi et al., [35] performed the FEMU of the bell tower using the GA for the automatic calibration of the elastic parameters to reduce modelling error following the model assurance criterion. In addition to automated FEMU, the authors performed also the manually calibration. Comparing the results obtained using both FEMU methods, it was concluded that with the GA the numerical model was much improved, but the frequencies had a higher deviation. The obtained results of comparison authors connected with the possible influence of soil structure interaction. To obtain an accurate and robust numerical model of the Baptistery of San Giovanni in Firenze, Lacanna et al., [41] performed FEMU based on Genetic algorithm using ambient vibration test data. Jiménez-Alonso and Sáez [174] used GA to performed the FEMU and help understand the actual state of structural conservation of the reinforced concrete truss bridge to select the appropriate retrofitting technique. Costa et al., [175] calibrated a numerical model of stone masonry arch railway bridge with GA by using dynamic modal parameters estimated from an ambient vibration test. Sabamehr et al [176] used GA to identify system properties and find correlations between the structural frequencies and changes in the sectional properties of the bridge segment. Pachón et al [177] used GA to adjust the numerical model of the E. Torroja's bridge to the experimental results with a reduced number of sensors. Hernández-Díaz et al., [178] used GA to obtain the numerical acceleration at the mid span of the footbridge under different pedestrian flows. Gentilini et al., [179] proposed a procedure based on dynamic testing, added masses and genetic algorithm (GA) to identify the tensile force, the modulus of elasticity of the material and the rotational stiffness of restrains for structural characterization of the tie rods.

Yang et al.[180] use GA, to solve the optimization problem of surrogate-based FEMU of a three-story structure using the frequency response function. Kim et al., [108] employed a FEMU method based on static and dynamic data sets to improve the identification of structural updating parameters. They used GA, to identify the updating parameters for the conventional FEMU method and verified the effectiveness of the proposed FEMU method on a simply supported plate girder prestressed bridge deck. Sun et al., [181] used a genetic algorithm to evaluate the Pareto-optimal solution of the updating parameters to perform fuzzy FEMU and accurately evaluate the mechanical state of an in-service cable-stayed bridge. Oh et al., [182] proposed a dynamic displacement based FEMU using a motion capture system to find parameters that minimize the difference between the updated model and the direct measurement. To minimize the combined error functions with the same number of modes simultaneously, non-dominated sorting genetic algorithm-II was used. Cui et al., [183] proposed a FEMU method of structural multi-scale monitoring model based on multiobjective optimization using the non-dominated sorting genetic algorithm- II (NSGA-II) is used to obtain the optimal parameter values of the large shell structure of Zhuhai Opera House. Wang et al., [184] used a non-dominated sorting genetic algorithm to perform a multi scale model updating of the structure of a transmission tower using the measured dynamic response as well as the static displacement and strain response. Luong et al., [185] used a GA, to update the steel truss structures using vibration-based data and identified the axial forces in all members. Sun and Xu [87] used a non-dominated sorting GA, to perform the FEMU of a long-span aqueduct structure based on the multi-objective optimization. Tran-Ngoc et al., [168] performed the FEMU for the bridge using GA and particle swarm optimization (PSO), and analysed and evaluated the effects of different joint assumptions on large-scale truss bridge. The comparison of the results obtained with both FEMU methods showed that the PSO algorithm provides a better result and reduces the computation time. In addition, the authors confirmed that the dynamic analysis results are extremely sensitive to the assumptions for the joints. To perform a preliminary evaluation of the bridge structure in terms of its mechanical resistance and stability after an earthquake damage, Mosquera et al., [186] used GA, to perform a high fidelity finite element modal updating.

In addition, GA is also used to update the finite element model for the purpose of damage detection. Srinivas et al., [101] identified and quantified damage to beams and plane truss structures by implementing a multi stage approach based on modal strain ratio change using GA. To minimize the differences between the analytical and experimental modal properties of a concrete-filled steel tubular arch bridge, Zhou et al., [85] used three artificial intelligence algorithms to calibrate uncertain parameters. These three algorithms were the simple genetic algorithm (SGA), the simulated annealing algorithm (SAA), and the genetic annealing hybrid algorithm (GAHA). The results of this study showed that the use of GAHA gave the best performance. In addition, the arch bridge could be efficiently calibrated using a combination of SGA and SAA. Park et al., [187] used GA, to solve the optimization problem of the proposed damage detection method based on FEMU. The proposed method is based on the modal participation ratio (MPR), which is defined as an indicator of the extent of modal contribution. This ratio was used to define the objective function as the differences between the MPR extracted from the sensors and the MPR estimated from the model. Jeenkour et al., [188] proposed encoding by locations (ELD) and damage factor and used GA to determine the location and extent of damage to the beam in which the location and damage extent were used as a decision variable in GA. Wang et al., [100] proposed a multilayer genetic algorithm for damage detection of truss structures to solve this problem. In the proposed method, the damage detection is divided into several groups for optimization purposes. Comparing the proposed method with GA, the advantages of the proposed method can be highlighted, such as computational efficiency, less possibility of local optima, and small size of search for each group.

In general, when updating the finite element model, the most important thing is to find a computationally efficient algorithm that can solve the proven FEMU optimization problem. For complex structures (cable-stayed bridges, suspension bridges and other complex structures), model updating with GA is very difficult to perform in terms of computation time and obtaining accurate results. For this reason, there are many studies in which the genetic algorithm is combined with another method to reduce the computation time and provide the

Table 4
Review of the using computational intelligence GA and its hybridization algorithms for FEMU, damage detection and optimal sensor placement.

Reported Application	Examples of related study	Type of GA	Structure
FEMU	Ye and Chen [4]	GA	High rise structure
	Jiménez-Alonso et al., [110]		Laboratory footbridge
	Bianconi et al., [35]		Bell tower of the cathedral
	Lacanna et al., [41]		Historical building (Baptistery)
	Jiménez-Alonso and Sáez [174]		Reinforced concrete truss bridge
	Costa et al., [175]		Arch Railway bridge
	Sabamehr et al., [176]		Pre-stress concrete box girder bridge
	Pachón et al., [177]		Inverted truss bridge
	Hernández-Díaz et al., [178]		Cable-stayed footbridge
	Gentilini et al., [179]		Tie rods in historical buildings
	Yang et al., [180]	GA (Surrogate based)	Three story structures
	Kim et al., [108]	GA (Sequential framework)	Simply supported plate girder prestressed bridge
	Sun et al., [181]	GA (Fuzzy FEMU)	Cable stayed bridge
	Oh et al., [182]	NSGA-II	Three story shear frame
	Cui et al., [183]		Large shell structure
	Wang et al., [184]		Transmission tower
	Luong et al., [185]	GA and strategy validation criteria	Truss structures
	Sun and Xu [87]	Non-dominated sorting GA	Long-span aqueduct structure
	Jin et al., [118]	GA and NSGA-II	Highway bridge structure
	Mosquera et al., [186]	GA of High Fidelity Finite Element model	Highway bridges
Damage detection	Boonlong [192]	CCGA	Beam
	Jeenkour et al., [188]	GA with ELD	
	Srinivas et al., [101]	GA	Beam, plane truss structure
	Zhou et al., [85]	SGA, SAA, GAHA	Concrete-Filled Steel Tubular Arch Bridge
	Wang et al., [100]	ML-GA	Truss bridge
	Park et al., [187]	GA and NSGA-II	Four story shear type building
Optimal sensor placement	Pachón et al., [193]	GA	Historical Masonry Tower
	Soman et al., [190]	GA	Aluminium Plate
	Hou et al., [191]	GA (Damage detection using L1 regularization)	Cantilever beam, 2-storey frame structure
	Nasr et al., [173]	GA in combination with the EnKF (Damage detection)	10-story shear building

required accuracy of the numerical model. Erdogan et al., [106] used GA, to solve the inverse fuzzy model updating problem of the benchmark test structure using static and dynamic data sets under different loads and conditions. Jin et al., [118] used GA to solve the single objective problem and NSGA-II to solve the multi-objective problem of FEMU of highway bridge structures using modal properties. Yu et al., [189] used GA, to perform the FEMU of the cable stayed bridge response surface model using structural health monitoring data. In addition, there are examples of research reporting the use of a genetic algorithm for optimal placement of sensors for structural identification and damage detection. Soman et al., [190] proposed the implementation of GA to improve the coverage of sensor networks for damage detection using guided waves. The first step of the proposed method was to determine the minimum number of sensors based on the quality of the signal processing algorithm. The second step involved determining the optimal sensor placement by improving the implementation of GA. Hou et al., [191] used GA to define the optimal sensor placement for determining the modal parameters used for L1-regulated damage detection of cantilever beams and three-story frame structures. Nasr et al., [173] proposed a method for optimal sensor placement by combining GA with the Ensemble Kalman filter for structural system identification and damage detection.

The literature review on finite element model updating under the GA optimization (Table 4) shows that this method is widely used and frequently applied in solving such problems as model updating, damage detection, and optimal sensor placement. This is mainly due to its ease of use and integration in the computational software, Matlab, most commonly used for model updating and solving optimization problems. On the other hand, the main problem is the computational effort required to solve the optimization problem. Several studies have compared the computational efficiency of GA with that of other optimization algorithms in solving single and multi-objective optimization problems, and it was found that it takes the most time.

6.2. Particle swarm optimization

Particle swarm optimization was introduced by Kennedy and Eberhart [194], inspired by the social behaviour and movement dynamics of animals and insects. It represents an efficient global optimization method for continuous variable problems that can be easily implemented with very few parameters. It is successfully applied in various types of optimization problems and in solving the FEMU optimization problem. The basic term in this method is the particle that stores the best position data it has ever visited, and the particle that was closest to the target in the whole particle swarm (global PSO-g_{best}) or only in its neighbourhood (local PSO-l_{best}), determined by its position and velocity. Based on the information the particle gathers, it decides on the speed of movement to the new position. Position, X_i^n, is the solution reached by the i-th particle out of a total of m swarm particles in the n-th iteration. Position is defined by coordinates in s-dimensional space, where s represents the number of variables, x_{ij}^n, that make up the solution $X_i^n = \{x_{i1}^n, x_{i2}^n, \cdots, x_{is}^n\}, i = 1, 2, \cdots, p$. The velocity of the particles is represented by the ratio of the position changes. A graphical representation of the PSO optimization algorithm can be seen in Fig. 4, where the main steps can be summarised as follows:

- Definition of the number of particles, initialization of the algorithm constants (position and velocity).
- Definition of an objective function as the difference between the current position and the target position.
- Recording and updating the best particle position and the best position ever reached by the swarm members

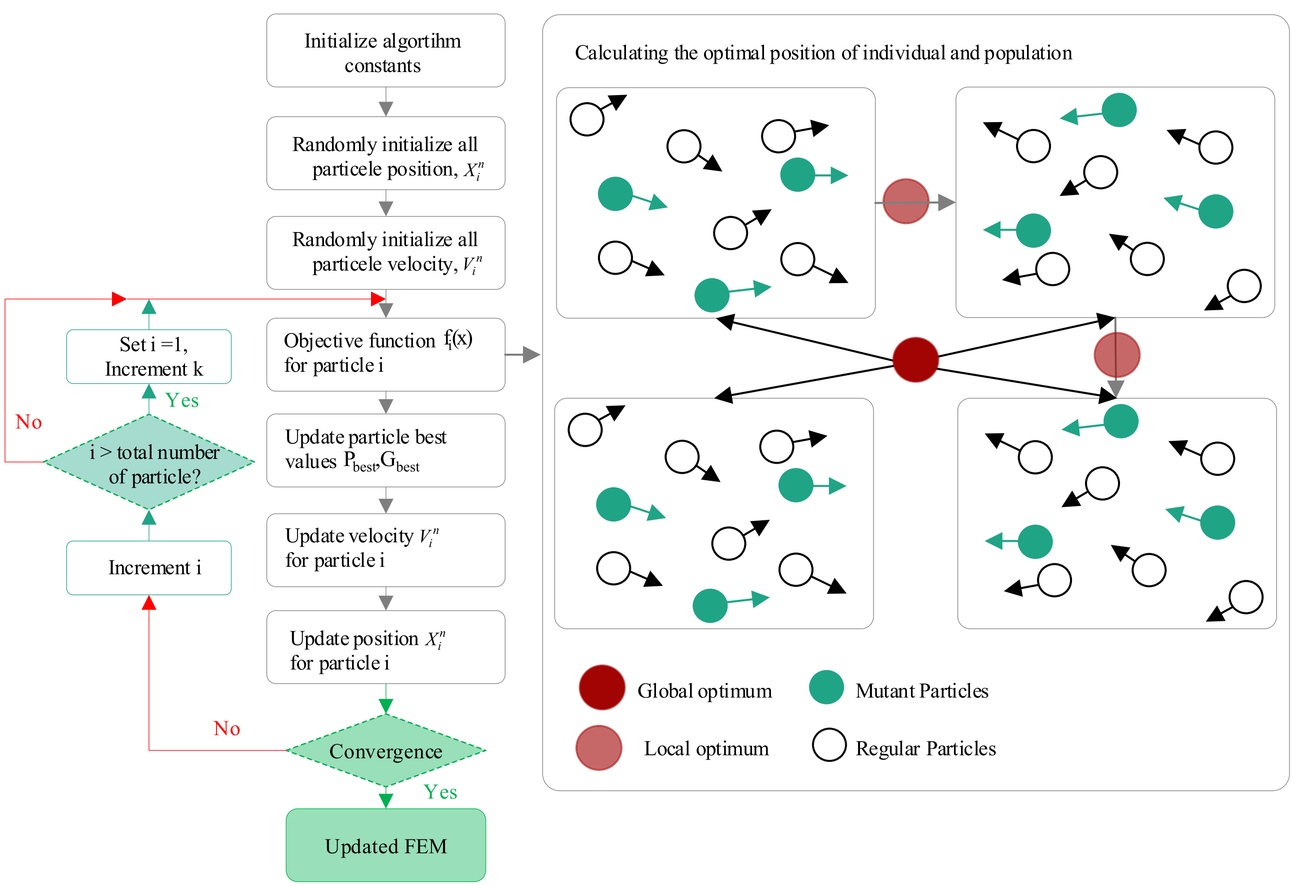

Fig. 4. Flowchart of Particle swarm optimization.

- Updating the velocity of the particle swarm according to the equation: $x_{ij}^{n+1} = x_{ij}^{n} + v_{ij}^{n+1}$, where v_{ij}^{n+1} is the velocity, x_{ij}^{n} is the position in the iteration n + 1, and x_{ij}^{n} is the position in the iteration n. The velocity is calculated with the next equation:

$$v_{ij}^{n+1} = wv_{ij}^{n} + C_1 rand_1 \left(Pbest_{ij} - x_{ij}^{n}\right) + C_2 rand_2 \left(Gbest_j - x_{ij}^{n}\right)$$

- In the previous equation, C_1 and C_2 represent learning factors. These factors are positive weighting coefficients used to balance the influence of individual and social experiences. $rand_1$ and $rand_2$ are random numbers between zero and one, while $Pbest_{ij}$ and $Gbest_j$ are the best positions achieved by the i-th agent closest to the target since the beginning of the process.
- Update the position of each particle based on social behaviour to match the environment by constantly returning to the most promising identified region.
- Repeat steps 1–5 until the termination criteria are met.

Tran-Ngoc [168] used particle swarm optimization (PSO) to update the model of Nam O Bridge using vibration-based SHM to build a reliable model for health condition assessment and operational safety management of the bridge. They mainly focused on the stiffness conditions of typical joints of truss structures and concluded that adopting semi-rigid joints (using rotational springs) can most accurately represent the dynamic behaviour of the truss bridge.

Many studies dealings with the application of the PSO algorithm in FEMU show that it can be used for both simple and complex damage detection models. Gökdağ and Yildiz [195] proposed a model update for damage detection of a Timoshenko beam using a particle swarm optimization algorithm using the modal parameters. Marwala et al., [196] used PSO to perform model updating damage detection of simply supported beam and H-shaped structures. The damage detection performed and the comparison with GA and Simulated Annealing (SA) showed that PSO has better performance. In addition, the combination of Nelder-Mead's simplex method (NM) and PSO has been shown to be very effective in damage detection, using PSO for global optimization and NM for local optimization. Jiménez-Alonso et al., [110] compared three optimization algorithms Harmony search, genetic algorithm and PSO by performing model updating of a laboratory steel footbridge. From the comparison, it was found that the accuracy of PSO and HS algorithm is similar and greater than that of GA. Mohan et al., [197] presented a robust finite element damage detection method using FRF as input response in the objective function and evaluated for beam and plane frame structures. Seyedpoor [198] proposed a two-stage method using modal strain energy and PSO to correctly detect structural damage in cases of multiple damage. In the first stage, the modal strain energy was calculated based on the modal analysis obtained from finite element modelling. Based on the data obtained in the first stage, the extent of damage was determined in the second stage using the PSO algorithm. The success of the method was investigated on a cantilever beam and a planar truss. Nanda et al., [199] used the particle swarm optimization algorithm to identify damage in a frame structure by varying the flexibility and modal data. Zhang et al., [200] proposed a FEMU method for damage detection based on multivariable wavelet FEM method and PSO optimization. In the first step of the proposed method, the multivariable wavelet FEM method (MWFEM) was used to model the structure with the crack. In the second step, the values of the natural frequencies were obtained, which were combined using the PSO to determine the location and size of the crack. Gerist and Maheri [201] introduced a two-stage, structural damage detection using PSO optimization. In the first stage, preliminary identification of structural damage is performed using sparse recovers. The results obtained in the first stage are improved to the exact location and extent in the second stage using the micro-search (MS) embedded in the PSO search. The effectiveness of the proposed method has been tested on several different types of the model. Nouri Shirazi et al., [202] proposed an adaptive multi-stage particle swarm optimization (MPSO) method to detect damage based on changes in natural frequencies. In the proposed method, the PSO deals with real values of damage variables. Perera et al., [203] proposed a method to solve the multi objective finite element model for damage detection on frame-like structures when modelling errors. First, the formulation of the objective function was developed using the modal parameters sensitive to modelling errors. Then, multi objective PSO (MOPSO) was applied to a multiobjective framework.

Table 5
Review of the using computational intelligence PSO and its hybridization algorithms for FEMU, damage detection and other purpose.

Reported application	Examples of related study	Type of PSO	Structure
FEMU	Tran-Ngoc [168]	PSO	Long span truss bridge
Damage detection	Gökdağ And Yildiz [195]	PSO	Timoshenko Beam
	Marwala et al., [196]		Beam and frame
	Mohan et al., [197]		
	Seyedpoor [198]		Beam and planar truss
	Nanda et al., [199]	UPSO (Unified particle swarm optimization)	Frame like structure
	Zhang et al., [200]	PSO with MWFEM	Beam
	Gerist and Maheri [201]	PSO in combination with the Basis pursuit de-noising method	Beam, Frame, Truss
	Nouri Shirazi et al., [202]	Multistage MPSO	
	Perera et al., [203]	MOPSO	
*	Cancelli et al., [204]	PSO	Bridge Girder
**	Mthembu et al., [66]		Unsymmetrical H beam
***	Chatterjee et al., [205]	PSO-NN	RC multi-storey building

*-condition assessment.
** -Selection the FE model.
***- Selection of the optimal weights for the Neural Network classifier.

Apart from using PSO for model updating and finding optimal values for unknown parameters and damage detection, other possibilities of the PSO algorithm have also been researched. Cancelli et al., [204] proposed a new approach to extract and analyse the vibration characteristics of the structure in order to obtain the condition assessment of the bridge girder. As part of the proposed approach, FEMU was performed using particle swarm optimization to fit the extracted reduce order stiffness matrix and modal properties. The proposed approach was found to be very effective in locating and quantifying damage along the beam with very high accuracy. Mthembu et al., [66] proposed the application of particle swarm optimization to the problem of selecting FEM, where an optimal model is the one that has the smallest number of updated parameters and the smallest deviation of the parameter variables from the mean material properties. To overcome the problem of local optimization and premature convergence of traditional learning algorithms, Chatterjee et al., [205] proposed a particle swarm optimization based approach for training Neural Network (NN-PSO). In this method, PSO was used to select the optimal weights for the neural network classifier. The proposed method was evaluated on a multi-storey RC building and was found to be very effective in predicting structural failure.

Previous studies which have dealt with PSO-based finite element model updating showed that it is a very efficient method to solve the optimization problem of finite element model updating. Moreover, it does not require any knowledge of the function, or its derivatives and it can explore multiple possible solutions in parallel manner in the same sequence. Since it is a global method, its performance does not depend on the initial population solutions. Notwithstanding the foregoing, these methods have also their drawbacks. The first is related to the solution obtained, i.e. there is no guarantee that it is more optimal than the other solution, nor is there convergence to the overall optimal value. The second one is related to the definition of the parameters, because all other algorithms require the definition of parameters that ultimately affect the final performance.

The following table (Table 5) represents the sum of the previously mentioned research and studies on the implementation of the finite element model update for damage detection, condition assessment, model selection, selection of optimal weights, etc., using the different types of particle swarm optimization algorithms.

Previous studies dealing with PSO-based finite element model updating have shown that it is a very efficient method to solve the optimization problem of finite element model updating. Moreover, it does not require any knowledge of the function or its derivatives and can explore multiple possible solutions in parallel manner in the same sequence. Since it is a global method, its performance does not depend on the solutions of the initial population. Notwithstanding the above, these methods also have their drawbacks. The first one is related to the obtained solution, i.e., there is no guarantee that it is more optimal than the other solution, nor is there convergence to the overall optimal value. The second one is related to the definition of the parameters, because all other algorithms require the definition of parameters that ultimately affect the final performance.

6.3. Other optimization algorithms

In addition to the aforementioned computer intelligence optimization algorithms, GA and PSO, which are due to their availability in computational software, most commonly used, there are a number of algorithms developed for the purposes of FEMU. Most of them are nature inspired, such as harmony search [206], simulated annealing [207], grey wolf [208], colliding bodies [209], gravitational search [210], and several others [211]. In this paper, only some of them will be discussed and presented in detail, due to the simplicity, such as simulated annealing and harmony search. For some other nature inspired algorithms' researches and studies, readers are referred to the following reference [211].

6.3.1. Simulated annealing

Simulated annealing is an optimization method proposed by Kirkpatrick et al., [207]. It is a probabilistic algorithm used to approximate the global optimum of a function by searching for the global extrema of a constrained function around a certain configuration range. The basic concept of this approach comes from annealing in metallurgy. The metal is slowly heated and cooled under controlled conditions until the desired properties are achieved. The process of controlled slow cooling is called annealing. Through a cooling process that promotes diffusion, the metal progresses toward a state of thermal equilibrium, reaching a state of minimum energy. Rapid cooling keeps the metal in a metastable state and prevents the metal's phase transition. Metropolis et al., [212] described how to simulate a group of molecules in thermal equilibrium at a constant temperature *T*. In a simulation, a randomly selected molecule is randomly shifted, after which the energy change ΔE [J] of the entire group is calculated. The most important result is the

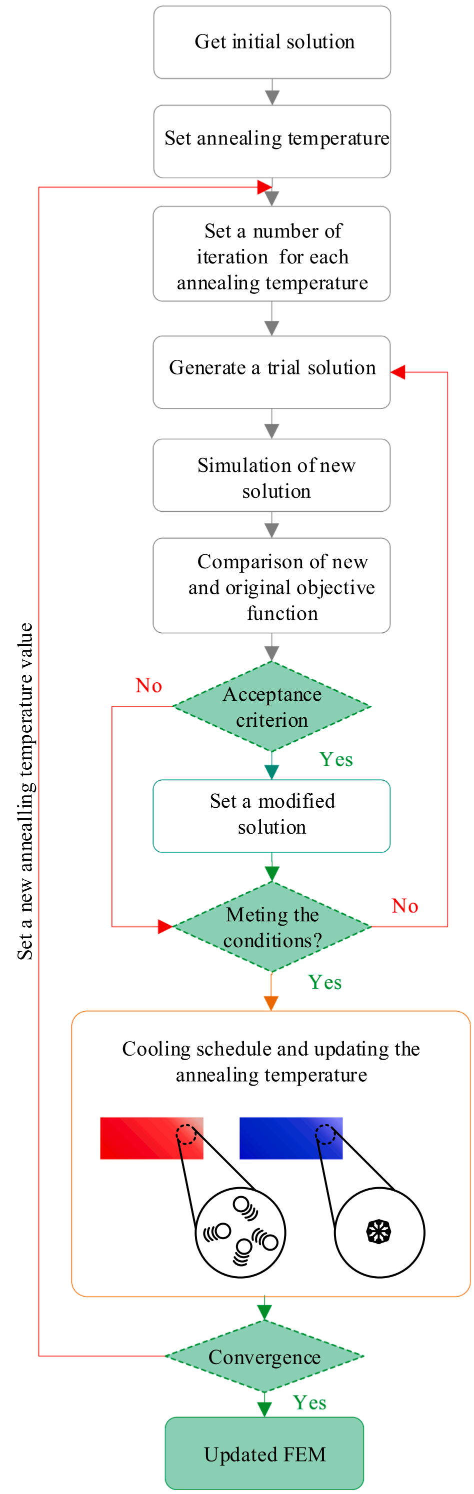

Fig. 5. Flowchart of Simulated annealing optimization algorithm.

Metropolis acceptance criterion, which defines the probability of acceptance of a simulated energy change, P_r.

$$P_r\{\Delta E\} = \begin{cases} 1, \& \Delta E \leq 0 \\ e^{-\Delta E/K_B T}, \& \Delta E \geq 0 \end{cases} \quad (17)$$

According to the above equation (17), if the energy gain is negative the total energy of the system is accepted. On the other hand, if the change increases the total energy of the system, then it is accepted with the probability $e^{-\Delta E/K_B T}$, where K_B is the Boltzmann constant. If the simulation is performed for a sufficient number of random motions, the final arrangement of the molecules is close to that in thermal equilibrium or steady state. This is the global minimum at temperature *T[K]*. The formal proof of convergence models the described simulation as a homogeneous Markov chain whose steady state is shown to correspond to thermal equilibrium. Theoretically, convergence is achieved only for an infinite number of simulations. The three assumptions under which thermal equilibrium is achieved according to the Metropolis algorithm are (1) Reversibility (symmetry) - the probability of choosing the next state is the same as the probability of returning from the next state to the current state (2) Ergodicity - the random displacements of the molecules are such that the molecules can reach any position in their configuration space (3) Convergence to the canonical distribution - the probabilities in the acceptance criterion are such that the ensemble weighs on average in the Boltzmann (Gibbs) distribution. The simulated cooling process is performed using the Metropolis algorithm which is performed through following steps: (1) Melt a system optimized to a high temperature; (2) At very high temperatures, all energy states are almost equally likely; (3) Slowly lower the temperature until the system freezes and there are no more changes; (4) At each temperature, the Metropolis simulation must be run until the system reaches a steady state at that temperature. The process of the simulated annealing optimization algorithm is shown on Fig. 5.

Levin and Lieven [210] performed a comparison of two powerful optimizations genetic algorithms and simulated annealing- to perform the model updating in the frequency domain. Based on the performed comparison they found that the SA is better than traditional GA and that the accuracy of model updating is dependent on the appropriate selection of the updating parameters. Marwala [211] has done the comparison of computational efficiency of Simulated Annealing, genetic algorithms and Response Surface method on an H-shaped structure. The comparison shows that the response surface method is 2.5 times faster than the genetic algorithm and 24 times faster than simulated annealing. Zhou et al., [82] presented ambient vibration measurements to develop a baseline modal for a newly constructed arch bridge over rives using the structural dynamic properties and performing the updating of numerical model with three algorithms including the simple genetic algorithm, the simulated annealing algorithm and genetic annealing hybrid algorithm. The SAA converged on an infeasible design because it began from a random point and then worked its way toward the minimum, meaning that a local minimum is more likely to be reached. Kourehli [212] proposed a damage detection method based on the simulated annealing using 3 different objective functions based on static and dynamic measurements, which is verified on a four-story steel frame (IASC-ASCE benchmark structure).

Some of the other authors work on the improving the computation efficiency of the finite element model updating process under the simulated annealing. Zimmerman and Lynch [213] increased the computational efficiency of SA, by dividing the annealing into a series of steps, each of which is executed on each computer node. The efficiency of the algorithm was tested on three-story structures, and it was found that the larger number of sensors in the network resulted in efficiency gains. There are studies in which the simulated annealing is used in combination with stochastic Bayesian model updating. Thus, Lam et al., [214] used the simulated annealing to propose a Bayesian finite element model updating damage detection based on structural dynamic properties. Huang et al., [215] use simulated annealing to obtain maximum a posteriori values of posterior PDF of design variables for characterizing damage and quantifying uncertainty. Green [216] proposes in his work a new MCMC algorithm: “Data Annealing”, which is based on the input of the likelihood of training data, so that its effects on the posterior are introduced gradually. Moreover, the proposed approach reduces the computational effort probability that local search will get stuck in local traps. Chiu and Lie [19] developed an algorithm based on simulated annealing to cope with the problem of finding the optimal sensor placement under the minimum cost limitation.

Based on the literature review conducted on the use of Simulated Annealing (Table 6) to perform model updating and closely related processes such as damage detection and optimal sensor placement, it can be highlighted that this method has not received as much attention as other computational optimization algorithms such as GA and PSO. This is mostly due to the SA requirements of a large number of annealing cycles and a slow convergence speed. These problems question the limitations of the applicability of SA to complex types of structures. However, it can be seen in the literature that authors are making efforts to solve this problem and combine the advantages of SA to develop some hybridization optimization algorithms by taking advantage of SA, such as its strength in solving combinational problems and good performance in hill climbing for the optimal solution.

6.3.2. Harmony search

Harmony search [221] is an optimization algorithm proposed by Geem et al., [222], primarily intended to imitate a simplified model of improvisation where there are no chords or modes, only notes or tones. Each tone represents a value of a design variable, and each musician represents a design variable. The vector for optimizing a particular objective function forms the entire harmony. Given the notes that the musician has already played, he chooses a new note to change the harmony. These changes can also be made by pitch or by playing an adjacent tone. The harmony search algorithm consists of three steps. In

Table 6
Review of the using computational intelligence SA optimization algorithm.

Reported application	Examples of related study	Type of simulated annealing optimization	Structure
FEMU	Levin and Lieven [213]	SA and GA	2D cantilevered beam
	Marwala [214]	SA, GA, RSB	Unsymmetric H shaped structure
	Zhou et al., [85]	SG, SA, GA	Steel tubular arch bridge
	Haung et al [218]	SA	IASC-ASCE Phase II benchmark problems
	Kourehli [215]	SA	IASC- ASCE
	Lam et al., [217]	Bayesian FEMU (SA for sampling scheme)	Shear building model under laboratory condition
Improving computational efficiency	Green [219]	Data Annealing	Nonlinear dynamical system consist of aluminium rod with centre magnet and two outer magnet
	Zimmerman and Lynch [216]	Parallel SA	Three story steel structure
Optimal sensor placement	Chiu and Lie [220]	SA	Smaller and larger rectangular sensor fields

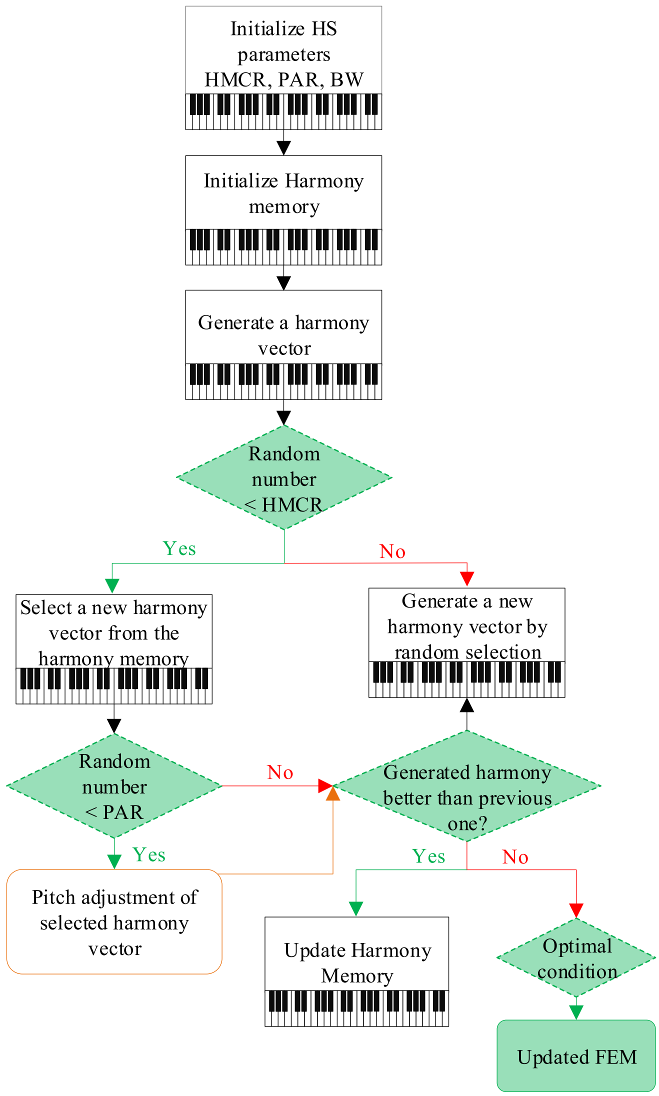

Fig. 6. Flowchart of Harmony search optimization algorithm.

the first step, a random population of possible solutions is created, which is stored in the Harmonic Memory Matrix. In the second step, an objective function is evaluated for each of the possible solutions. In the third step, a new harmony is created at each iteration, the maximum number of which is determined by the maximum improvisation parameter (MI). The evaluation of the objective function for each new harmony is performed in the fourth step. By comparing the original and the new harmonies, the harmony memory matrix is updated in the fifth step. Until the convergence criteria are met, steps 3–5 are repeated. For the third step and the development of a new harmony, three mechanisms can be used. These include random selection, memory selection, and pitch adjustment [206]. Each design variable of the new vector can be determined from previous values stored in the harmony memory matrix HM or from a new random value. The probability of selecting the previous element of the harmony memory matrix is determined by the Harmony Memory Consideration Rate, HMCR. When the value is taken from the previous value, it changes according to the pitch adjustment rate, PAR, taking into account a predefined range of possible values. The complete process of harmony search is shown in the Fig. 6. This optimization process is characteristic of single-objective optimization, while in multi-objective optimization, both the harmony memory consideration and the pitch adjustment rate are used in each iteration to define a new value for the design variables. To rank the solutions of multi-objective optimization, non-dominant sorting [223] and crowding

Table 7
Hybridization of the finite element model updating genetic algorithm.

Reported Application	Examples of related study	Type of GA	Structure
FEMU	Jung and Kim [229]	HGA	Simply supported bridge
	Qin et al., [169]	GA in combination with Kriging model	Tied basket arch bridge
	Tran-Ngoc et al., [168]	SGA, GAHAA	Steel truss bridge
	Shabbir and Omenzetter [172]	GA and sequential niche technique	Cable stayed footbridge
Damage detection	Boonlong [192]	CCGA	beam
	Cha and Buyukozturk [113]	HGA	3D steel structures
	Shallan et al., [230]	HGA + SQP + Inter point + Active Set	Beam and Frame
	Raich et al., [170]	IRR GA	Beam and Frame

distance [224] are used.

As for the application of the Harmony search algorithm in solving the finite element model updating optimization problem and the related global problem, there are few studies [119,225-228] in which it is used for this purpose. Most of the papers [119,226,227] focus on combining harmony search with other algorithms to improve the computational efficiency. When compared to the traditionally used optimization algorithms (GA, PSO) [110] as standalone algorithms, it can be seen that HS is the most efficient among them when comparing the computational cost and accuracy of the adjustment. This subsection has served only as introduction for mathematical background and description of steps of the harmony search algorithm, while its application and examples of studies will be more discussed in the following section.

6.4. Hybrid local–global optimization algorithms

As can be seen from the previous subsections dealing with the implementation of model updating and closely related processes such as damage detection, the use of the discussed algorithms as stand-alone algorithms is very computationally intensive due to the solution of very complex mathematical problems. Moreover, the search for solutions can often get stuck in local traps. To solve the above problems, many studies and research have proposed (Table 7, 8 and 9) hybrid local–global algorithms that can successfully solve this problem by combining the advantages and disadvantages of the standalone algorithms.

As for the Genetic algorithm, their hybridization in order to perform model updating more efficiently, Jung and Kim [229] proposed a hybrid genetic algorithm (HGA) by combining GA with Nelder-Mead's modified simplex method to improve the FE model of the bridge structure. Using a Kriging model, Qin et al., [169] proposed a hybrid algorithm to perform the FEMU of complex bridge structures. To increase the chance of finding the global minima and finding the minimum that best describes the system Shabbir and Omenzetter [172] used a combination of genetic algorithms and sequential niche technique which was tested to

Table 8
Hybridization of the finite element model updating particle swarm optimization algorithm.

Reported application	Examples of related study	Type of PSO	Structure
FEMU	Shabbir and Omenzetter [231]	HPSO-NT	Pedestrian cable stayed bridge
Damage detection	Luo and Yu [232]	PSO based sparse regularization	Beam
	Vakil-Baghmisheh et al., [233]	PS-NM	
	Saada et al., [234]	Modified PSO	
	Jena and Parhi [233]	MPSO	
	Kang et al., [235]	IEPSO	Beam, frame, truss
	Kaveh et al., [227]	HPSO	
	Alkayem et al., [171]	PSO and a social version of sine–cosine optimization algorithm	3D irregular shape structure
Time-variant reliability-based design optimization	Li and Chen [236]	PSO-t-IRS	Simply supported beam, two bar frame, six-bar indeterminate truss structure, 23-bar truss structure.

perform FEMU of simple laboratory structures and a full-scale pedestrian bridge.

To deal with the damage detection, Boonlong [192] proposed a cooperative coevolutionary genetic algorithm (CCGA) capable of solving an optimization problem with a large number of decision variables, as an optimizer for beam damage detection. In the proposed method, each population contains several types of subpopulations. Using the populations, the proposed method explores the search space with a smaller number of generated solutions. As in the classical genetic algorithm, each species is independently involved. Cha and Buyukozturk [113] proposed a damage detection approach using a multi objective hybrid genetic algorithm based on MSE to determine the exact location and extent of damage in 3D steel structures. Shallan et al., [230] used Hybrid GA in combination with sequential quadratic programming, interior point, and active set to minimize the objective function, and performed the localization and quantification of damage to beams and simple frames using static datasets from a limited number of sensors. Raich et al [170] presented the FRF-based damage detection method using the implicit redundant representation (IRR) GA, which identifies both the location and severity of the damage using the limited amount of measurement information. The effectiveness of the proposed method was demonstrated on cantilever beams, two-span continuous beam, and frame structures. Since the application of the optimization-based approaches to damage detection is slow to converge and requires a large

Table 9
Hybridization of the finite element model updating simulated annealing and harmony search optimization algorithms.

Optimization algorithm	Reported application	Examples of related study	Type of simulated annealing optimization	Structure
Particle swarm optimization	Damage detection	He and Hwang [237]	New hybrid algorithm combining the GA and SA	Damaged clamped beam, mild steel truss
		Rong and Shun [238]	Hybrid algorithm that combines the GA and SA	Helical spring optimization design case, auto regressive and moving average exogenous model
		Astroza et al., [239]	Hybrid global optimization algorithm (SA + UKF)	3D 5-story 3-by-2 bay steel frame building
Harmony search	FEMU	Lee and Geem [225]	Harmony search *meta*-heuristic	Different type of objective function Pressure vessel design Welded beam design 10/18 bar plane truss
		Naranjo-Pérez et al., [119]	Hybrid Unscented Kalman Filter-Harmony Search	Laboratory footbridge
		Naranjo-Pérez et al., [226]	Collaborative machine learning algorithm	Bormujos footbridge
	Damage detection	Kaveh et al., [227]	Particle swarm-ray optimization with harmony search (HRPSO)	Five story and four span frame, 52 bar space truss

amount of computation.

The Particle swarm optimization algorithm is also successfully used to propose a hybrid local–global optimization algorithms to solve, most often, FEMU and damage detection method. As a combination of GA and sequential niche technique, Shabbir and Omenzetter [231] also proposed a method that combine sequential niche technique with PSO. In this way, the authors made a possible a systematic search for multiple minima and confidence in finding the global minima was increased. In order to solve the damage detection problem, Luo and Yu [232] proposed a sparse regularisation method based on particle swarm optimization to detect structural damage. The proposed method consisted of two steps. In the first step, the FEM is updated based on the sensitivity analysis while the damage location is determined by the PSO. In the second step, the possible damage location and PSO are used to determine the extent of damage in subsequent iterations. Numerical simulations on a cantilever beam show the robustness and applicability of the proposed method. Vakil-Baghmisheh et al., [233] used a hybrid particle swarm Nelder Mead algorithm to perform damage detection in a cantilever beam by minimizing the objective function based on the differences in natural frequencies. Saada et al., [234] proposed a modified particle swarm optimization algorithm for damage detection in beam structures to facilitate the performance of FEMU in accordance with experimentally determined natural frequencies. The main idea of the modified method was to identify multiple optima by running the algorithm a predetermined number of times, each time identifying an optimal location. Jena and Parhi [233] modified the PSO technique (MPSO) with the strategy of squeezing the physical domain of the search space to perform an inverse analysis of damage identification based on natural frequency. Kang et al., [235] improved a PSO by combining it with the artificial immune system and developed a new immunity-based particle swarm optimization (IEPSO) algorithm for model updating damage detection. Compared with the classical PSO algorithm and various evolutionary algorithms, the proposed method showed better performance in determining the location and extent of damage to simply supported beams and truss structures. Kaveh et al., [227] proposed a mixed particle swarm ray optimization combined with harmony search for model updating damage detection of the 3D structure of a five-story frame and space truss structure. To solve the damage prediction problem for structures with irregular shape, Alkayem et al., [171] combined PSO and the sine–cosine (SCO) optimization algorithm and developed a new hybrid optimization algorithm. Using the social interaction between PSO and SCO, the highly nonlinear and multimodal optimization problem of FEMU -based damage detection was overcome. The reliability of the developed approach was tested on irregularly shaped 3D modular structures and proved to be very effective and efficient. Li and Chen [236] proposed a PSO-t- IRS to study time-varying reliability-based design optimization problems, which are also associated with high computational cost and difficulty in modelling. This method combines the PSO and the enhanced instantaneous response surface (t- IRS). Enhanced instantaneous response surface was used to construct the extended surrogate model for the instantaneous response, while the PSO was integrated with the extended surrogate model and used to find the optimal solution for the time-varying reliability-based design optimization. The effectiveness of the proposed approach was demonstrated in several examples, including a simply supported beam, a two-bar frame, a six-bar indeterminate truss structure, and a 23-bar truss structure.

The other nature inspired computation algorithms advantages are also used to propose and develop some better, more computational effective hybrid local–global optimization algorithms. Thus, He and Hwang [237] combined Genetic algorithm and Simulated annealing in order to propose a new hybrid algorithm for finding actual damage. The results of the validation of the proposed hybrid method showed its efficiency when the measured data are free of error. When the measured data have an error that is acceptable, the proposed method provides less accurate but still acceptable and reasonable results. In order to increase the quality of the solution and the speed of convergence, Rong and Shun [238] proposed a new algorithm that combines the advantages of genetic and the simulated annealing algorithm. The efficiency was tested on a helical spring optimization design case and system identification problem described by auto regressive and moving average exogenous model. Astroza et al., [239] combine simulated annealing with the unscented Kalman filter in their work to reduce the computational cost. The results show that the proposed combination saves significant computational time without affecting the estimation performance.

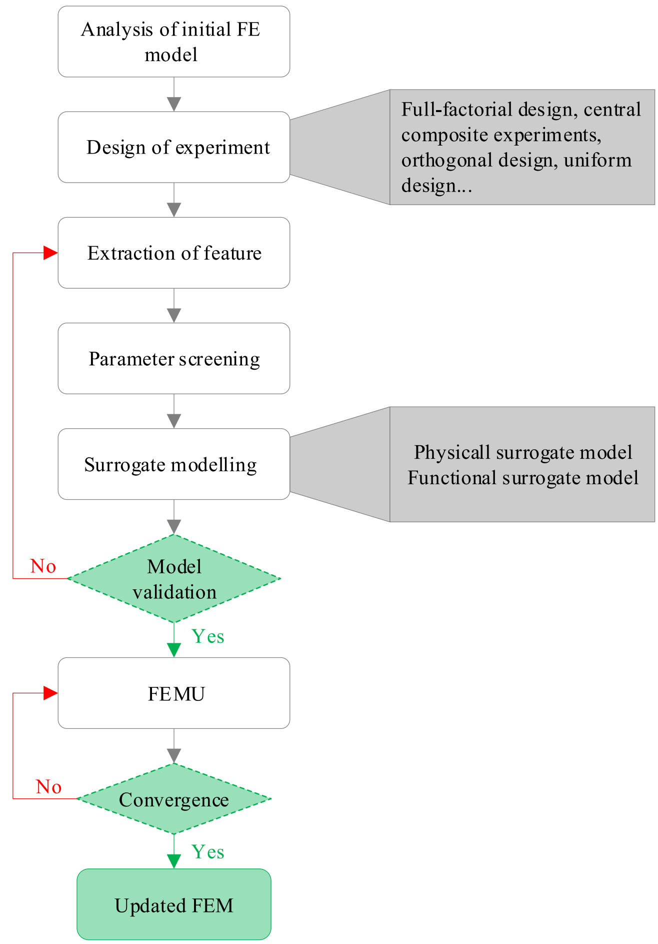

Fig. 7. FEMU using surrogate based method flowchart.

To reduce the high computational time for model updating of complex structures under the harmony search optimization algorithm, Naranjo-Pérez et al., [119] proposed a novel hybrid Unscented Kalman Filter- Harmony Search (UKF-HS). The performance of the proposed algorithm was tested on the benchmark footbridge under the context of single and multi-objective optimization and compared with three computational optimization algorithms. In another work, Naranjo-Pérez et al., [226] combine multi-objective harmony search, active set algorithms, artificial neural network and principal component analysis to solve the problem of high computational time and uncertainties associated with selecting the best updated model among all Pareto-optimal solutions. Kaveh et al., [227] proposed a new optimization algorithm for damage detection that combines mixed particle swarm ray optimization with harmony search. Miguel et al., [228] proposed a new vibration based method that combines a time-domain modal identification technique with the evolutionary harmony search algorithm. The proposed method was verified on a numerical example and three cantilever beams with different damage scenarios and noise levels. The results show that the proposed method can be efficiently used for structural damage detection and remaining life prediction.

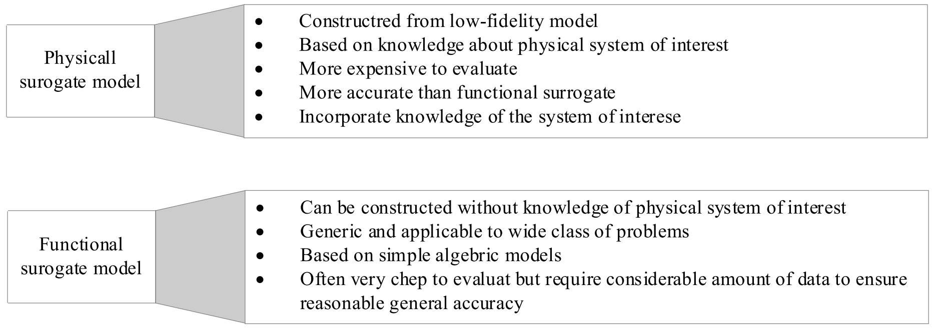

Fig. 8. Classification of the surrogate based models and their characteristics and main advantages.

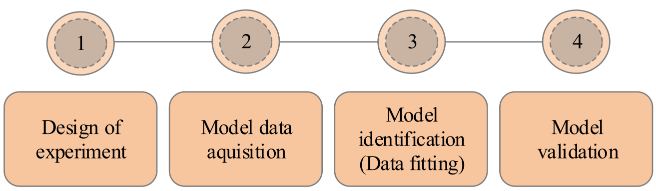

Fig. 9. Surrogate model construction flowchart.

7. Surrogate based finite element model updating

To reduce computational efficiency in finite element model updating problems, the finite element model was replaced by a mathematical model that approximates the relationship between the preselected inputs and outputs of the FE model. The main objective in developing the surrogate-based model updating method is to replace the original finite element model in order to obtain a surrogate that is analytically more practical and computationally more convenient. The use of surrogate-based optimization (SBO) replaces direct model optimization with an iterative process of creating, optimizing, and updating a fast and analytically tractable surrogate model. A locally defined surrogate model should have at least reasonable accuracy in representing a numerical model. By optimizing the design of the surrogate model, a design is obtained that is verified by evaluating a numerical model, and its data is further used to update the surrogate model. Until the termination criterion is satisfied, the optimization process continues by applying a predictor–corrector [240]. Performing all operations on a defined surrogate model reduces the duration and computational effort of the optimization compared to direct optimization. The procedure for performing surrogate-based model updating is presented below (Fig. 7). In the first step, an initial surrogate model is generated. In the second step, an approximate solution to the nonlinear minimization problem is obtained by optimizing the surrogate. Then, the model is evaluated based on the approximate solution computed in the previous step. In the fourth step, the new model is used to update the surrogate model. If the termination criteria are satisfied, the procedure is terminated. If the termination criteria are not satisfied, we proceed to the second step of the procedure.

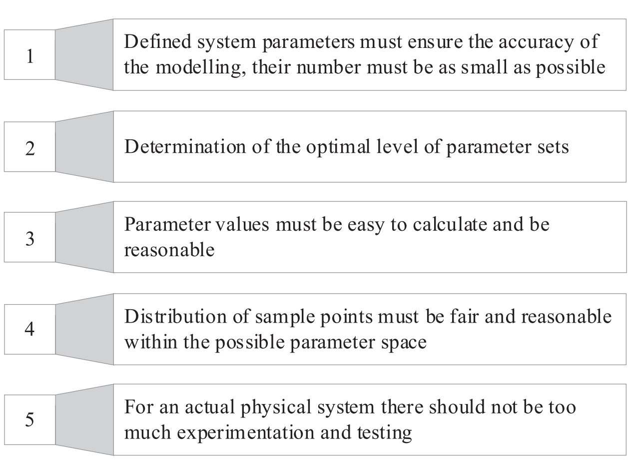

Fig. 10. Five requirements that design of experiment methods should fulfil.

Main terms in surrogate-based optimization are surrogate models and surrogate based optimization technique (Fig. 8). For developing the surrogate model design experiments, surrogate modelling technique, model validation and surrogate criterion are necessary. Surrogate based optimization technique include the approximation model management optimization, space mapping, manifold mapping, and surrogate management framework. Surrogate model is a key component of surrogate-based method, and it can be classified as the physical and functional surrogate models [241]. Surrogate model developing include strategy of design of experiments, model data acquisition, data fitting and model validation (Fig. 9).

Design of the experiment (DOE) is a strategy of assigning samples (points in the design space) that aim to maximise the amount of information collected. Model estimation is performed at the assigned points in the design space to create a data set that is later used to construct a surrogate model. To explore a large region of the search space, classical DOE techniques such as factorial designs [242] applied to discrete design variables are used. By applying samples of possible combinations, called factorial design, once discretized continuous variables can be easily analysed. In a fully factorial design, the number of samples increases exponentially with the number of design variables. When the number of design variables is large and model evaluation is expensive, fractional factorial designs [243] are used. In addition, to evaluate the effect and interaction among design variables, their quadratic effects and interactions, full two-level (2^r) and three level (3r) design is applied such as a central composite design [106], star design [244] and Box Behnken design [245]. During the construction of the initial surrogate model, there is usually non prior knowledge about the objective function. Therefore, some of the DOE approaches mainly aim at uniform distribution of samples within the design space [246]. Therefore, Latin hypercube sampling (LHS) [247-250] is mostly used. In this approach, the design space is divided into b^v bins, where b is the number of samples and v is the number of design variables. The samples are selected according to the following criteria: (1) each sample is within a bin, and (2) in each bin there is exactly one sample for all one-dimensional projections of the p samples and the bin. The standard LHS may lead to non-uniform distribution. Therefore, approaches that provide a uniform distribution of samples have been proposed. There are several other approaches to sampling, including Monte Carlo sampling [251], Hammersley sampling [132], and orthogonal array sampling [246].

The common DOE method used in the RS method refers to full-factorial experimental design [169], central composite experiments [252], orthogonal design [183], uniform design [253], etc. For low-order

RSM, orthogonal and uniform design are usually used. Full factorial experiment design requires too many calculations, although it could lead to relatively more accurate results. Central composite experiment [254] design and D-optimality [255] are applied to model updating of large RSMs, and they achieve almost the same accuracy as when a polynomial response surface is created [254]. Regardless of the DOE method chosen, the ideal DOE should fulfil the requirements shown in Fig. 10.

After the design of the experiment has been appropriately selected and the data collected, an approximate model and fitting methodology are established below.

The most popular surrogate modelling techniques include polynomial regression [252,256], radial basis function [252,257,258], Kriging predictor [184,249,259,260], neural network [261-263] and other methods based on the above [264-267]. For additional descriptions of previous methods, the authors refer the reader to some of the following references [264,266,268,269]. Some of these methods define a surrogate model and an estimate of the approximation error built into the process. These include the Kriging or Gaussian regression methods. Separately, there are methods that are used only to assess the predictability of a particular model. One of the simplest methods for validating a model is the split sample method [246]. In this method, the sample is divided into two subsets: a training subset and a testing subset. The training subset contains the points from which the surrogate model is created. On the other hand, there is the cross-validation method [270]. In this method, the data sets are divided into L subsets. Each subset is used in turn to test the surrogate model developed based on the other L-1 subsets. In the case where the number of subsets L is equal to the sample size p, the cross-validation is called leave-one-out cross-validation [271]. The advantage of this approach is that it is less biased compared to the split-sample method. However, the main disadvantage is that it requires the construction of a surrogate model. Nevertheless, the robustness of the surrogate model and the validation approach can be improved since all data are used simultaneously as training data and as test data.

7.1. Response surface-based method

In order to avoid model updating being trapped in local solutions that yield an unacceptable value of the objective function, in the initial phase of the surrogate-based model updating method, it is recommended to use the surrogate model that is valid in the global search space [272]. Therefore, regardless of the method used, a correction of the surrogate model is performed to avoid the possibility that the global accuracy of the model may be useless for the further optimization process. To improve the surrogate locally, two methods are used. The first one refers to the correction of the objective function [273] and the other one refers to the concept of space mapping concept [274]. The following is an example of studies in which some of the surrogate-based methods mostly the response surface based method applied to perform model updating, damage detection, or reliability analysis.

Ren et al., [253] proposed a FEMU based on the RSB method using measured static structural responses. In addition, a technique to reduce the parameters for constructing the RSB model was proposed. The method was verified on a numerical beam and a full-scale continuous box girder bridge. It was found that the proposed method has the advantages of simple implementation, highly efficient cost, reasonable updating accuracy, and does not require FE calculation in each iteration of the optimization procedure during updating. Ren and Chen [154] proposed an application of the response surface-based finite element model updating for the simulation of a simply supported beam and a full-size continuous box beam, and discussed the main aspects of its implementation. Using simulation data from FEM, a quadratic polynomial response surface was constructed and used to reduce computational costs. Comparison of FEMU with RSB using the sensitivity-based method showed that the RSB method is efficient and converges faster. The application of model updating using the response surface method is characterized by the difficulty of finding a suitable design, followed by a series of trials and errors with different dimes and subset models. To overcome this limitation, Shahidi et al., [245] proposed a generalized response surface based method. To extract as much data as possible from the measured signals and to compensate for the error that often occurs in regression models, the formulation of the MU problem in the time domain was also proposed. The efficiency of the proposed method was demonstrated on a steel frame structure. Marwala [214] presented a finite element model updating method in which the response surface equation of the finite element model is approximated by a multilayer perceptron. The updating parameters were determined by optimizing the objective function using a genetic algorithm. Verification of the proposed approach was performed on an asymmetric H-shaped structure. It showed that the proposed approach is 2.5 times faster than the genetic algorithm and 24 times faster than simulated annealing. To increase the influence of the sample points near the prediction point on its prediction value and solve the coefficients of the response surface polynomial, Chakraborty and Sen [275] combined the moving least square methods and response surface methods. The effectiveness of the proposed method was verified on a 10-member truss and a masonry culvert. Zong et al., [254] presented an application of the RS method on model updating of a bridge structure. By implementing the third order polynomial function, the response surface model of the bridge was created. In their work, the authors considered important aspects for the implementation of model updating, such as experimental design, screening of parameters, construction of a high-order polynomial RS model, optimization methods, and verification of the accuracy of the RS model. Compared with the traditional sensitivity-based model updating method, the proposed method was shown to be more efficient and converge quickly. Zhou et al., [252] validated the effectiveness of the radial basis function (RBF) based response surface method where an RS model was constructed using quadratic polynomials. The model updating procedure was demonstrated on a theoretical structure, a laboratory-scale bridge model, and a real structure using static and dynamic test results and SHM data. The focus was on the selection of updating parameters and responses for updating. The results showed that RMS can be easily implemented for updating complex structures such as long span cable-stayed bridges. Yu and Ou [189] combined the substructure method with the response surface model updating method using SHM data to reconstruct the actual operating condition of the Aizhai suspension bridge. The time-domain based RSM was obtained by comparing the characteristics of the time-domain response of the tests and their FEM counterparts and establishing the functional relationship between the time-domain response and the structural parameters. The unknown structural parameters of FEM are modified by optimization calculations based on the response surface method. Deng and Cai [276] proposed the response surface based method combined with GA to update the bridge model to obtain the explicit relationship between the structural responses and the parameters from the simulation results. By using second or higher order polynomials, the RSM can model the curvature effects between the responses and the parameters that the sensitivity-based methods cannot represent. In the proposed method, the first step is to select the updating parameters. The experimental design for the selected parameters was optimized using RSM. The structural responses were selected according to the purpose of model updating, while the minimization of the objective function was performed using GA. Using the regression method, the RSF for structural responses can be obtained, while the objective function can be developed using the residuals between the measured and predicted responses from the generated RSF and optimised using GA to obtain the updated structural parameters. To investigate the accuracy of the RSM based on the time domain, Han and Yang [277] conducted an experiment on a simply supported beam where the frequency response and modal parameters were obtained. Han and Yang compared the results with those obtained by applying the FEMU based on the frequency response function and modal characteristics and concluded that the time-domain based RSM can reduce the computation time and improve the efficiency of the

FEMU and can be used for further damage detection and condition assessment. Kriging model prediction is a modelling method based on the Gaussian process, which has been shown to be compact and effective in solving optimization problems. Su et al., [278] developed a Gaussian Process (GP) surrogate model to assess probabilistic seismic performance of a pile-supported wharf structure. GP was used as a substitute for the computationally intensive model. In terms of computational cost, the approach based on the surrogate model GP is far superior to the brute-force MCS. To address the challenge from the perspective of the Kriging model, Shan et al., [279] proposed a new method to update the finite element model by combining the substructure with the response surface method. The effectiveness of the proposed method was verified on a laboratory-scale cable-stayed suspension bridge. Moravej et al., [280] integrated a modular Bayesian approach to structural reliability analysis and proposed a structural performance evaluation approach using Gaussian process surrogate models. By substituting the finite element model and the associated discrepancy function for the Gaussian process surrogate model, efficient calculation and comprehensive uncertainty quantification were ensured. The proposed method was tested on a large box girder bridge at laboratory scale in undamaged and damaged conditions. On this basis, FEMU has been shown to be a robust and computationally efficient tool for calibrating structural properties under uncertainty. Wu et al., [247] proposed an update of the finite element model based on the combination of the Kriging model and Latin hypercube sampling to perform the model update for different bridge types. The Kriging model was used as a surrogate model, while the Latin hypercube sampling was applied to the preselected samples defined to predict the relationship between the predicted and actual structural behaviour. Comparing the computational cost of the proposed method with that of the Genetic Algorithm, the proposed method shows better performance. To reduce the computational cost of the large number of iterations required for Bayesian model updating, Mao et al., [249] used the Kriging predictor to build the surrogate model of the suspension bridge. The Latin hypercube sampling method was used to generate the experimental data sets. In addition, the authors investigated four types of correlation functions: the Gaussian, exponential, linear, and spline functions. The comparison of the correlation functions showed that the spline and linear correlation functions generate a much larger validation error than the Gaussian and exponential functions, while the error of the Gaussian correlation function is smaller than that of the exponential function. Based on the performed model update of the cable-stayed bridge, it was proved that using the Kriging predictor to update the large complex structure reduces the computational cost and ensures the accuracy. Wang et al., [184] proposed a multi-scale finite element model updating method using the kriging metamodel as a surrogate for the multi-scale model to perform the finite element model updating of the laboratory model of a transmission tower. The review of the proposed method has shown that it can improve the accuracy of the multiscale model in both local and global structural response. Bassoli et al., [166] performed the update of the finite element model of a masonry tower using a surrogate-assisted differential evolution algorithm to reduce the computational cost. To further reduce the number of objective function evaluations, a filling sampling strategy was introduced in the applied algorithm. The accuracy in the optimal region through local and global exploration was increased by selecting candidate points. Based on the research conducted, it was found that a fully characterised structure can be used to achieve a consistent description of the dynamic behaviour of the structure using numerical modelling. Aruna and Ganguli [21] performed a model update on a cantilever beam to solve the problem of understanding a reaction surface based multi-fidelity model and quantifying the uncertainty associated with free vibrations. Based on the conducted study, it was found that the responses obtained with the multi-fidelity model are very close to the responses obtained with the high-fidelity model. Moreover, when compared with the Monte Carlo-based quantification of uncertainty, it was found that the multi-fidelity model requires little computational time and is as accurate as the high-fidelity model.

Qin et al., [169] combined the Kriging model with a genetic algorithm in a hybrid finite element model updating method. Based on sampled data regression, a Kriging model was developed between the updating parameters, frequencies and displacements. The analytical values of frequency and displacement in the objective function are predicted by the kriging model and solved by the genetic algorithm.

In addition to the previous study, where the surrogate model is used to update the finite element model, it can be successfully used for damage detection and reliability analysis. Fang and Perera [255] proposed a FEMU method for damage detection based on the RSB method and the optimal D-design using only the natural frequency values. The authors used the D-design to establish response surface models and perform the screening out non-significant updating parameters because it requires a smaller number of samples than a standard design such as CCD and allows for an irregular design space. The proposed method was verified on the numerical model of a beam, a RC frame, and a full-scale bridge, and it was concluded that the linear RS model proved to be suitable for the purposes of SDI. Bucher and Bourgund [281] proposed a new adaptive interpolation scheme that provides a factual and accurate representation of the system behaviour through the response surface. To obtain the desired reliability estimates, the response surface was used in conjunction with advanced Monte Carlo simulation techniques. The effectiveness of the proposed method was verified using two examples that included an SDOF system and a three-bay five-story frame. Das and Zheng [282] proposed a method for cumulative formation of the response surface proposed a cumulative response surface function method to perform reliability analysis of a stiffened plate structure, which is very time consuming. The proposed method includes three main steps: search, improvement, and verification. In the first two steps, the iteration process reduces the distance between the central point and a sample point. The third step, the verification step, is performed based on the sample points already obtained. The proposed method has been verified on several examples with a 3 and 12-story frame. To determine reliability and obtain moderate computation time, Gaspar et al., [248] combined a response surface model based on second-order polynomials with a first-order reliability method. By combining the adaptive interpolation scheme and the Latin hypercube sampling technique, an iterative definition of the response surface model was performed in the space of basic random variables that contribute most to the probability of failure. The case study was performed on plate elements, and the effects of considering constrained or restrained boundary conditions and corroded plate elements on the reliability analysis results were observed. Li et al., [258] proposed a sequential surrogate reliability method based on radial basis functions. In the proposed method, the optimization problem is solved iteratively to update a surrogate model of the limit state function. By using new points and updating the surrogate model, the surrogate model of the limit state function becomes more accurate in the important region that has a high probability of failure and at the boundary of the limit state function. The main objective of the optimization is to find a new point that maximizes the probability density function. The proposed method was verified on several numerical examples and showed the accuracy of the surrogate model in the important regions with a smaller number of samples. To estimate the failure probability in each iteration, Monte Carlos simulation was used to obtain a sequence of approximate failure probabilities. Li et al., [250] proposed a new instantaneous response surface method (t- IRS) for time-dependent reliability analysis. The proposed method does not need to build and update the surrogate models separately for each time node. It uses the expansion optimal linear estimation method to discretize the stochastic process into a set of independent standard normal variables along with some deterministic functions of time. The initial samples are then generated by applying Latin Hypercube Sampling (LHS). The corresponding response values are used to construct the Kriging surrogate model of the instantaneous response. To update the Kriging surrogate model, the active learning method is used until satisfactory accuracy is

Table 10
Review of the using surrogate based method for FEMU, damage detection and reliability analysis.

Reported Application	Examples of related study	Types of SB method	Structure
FEMU	Ren et al., [253]	RS based on the measured static structural response	Numerical beam, full-scale box-girder bridge
	Ren and Chen [154]	RS	Beam, full-size precast continuous box girder bridge
	Shahidi et al., [245]	GRSMU	Steel frame structure
	Marwala [214]	RS + GA	H-shaped structure
	Chakraborty and Sen [275]	MLSM - RS	10 member truss, masonry culvert
	Zong et al., [254]	A third order polynomial RS	Long span prestressed continuous rigid frame bridge
	Zhou et al., [252]	RS method based on the RBFs	Cable stayed bridge
	Yu and Ou [189]	RS method using SHM	Suspension bridge
	Deng and Cai [276]	RS + GA	Simply supported concrete beam, Prestressed concrete slab on girder highway bridges
	Han and Yang [277]	RS based on time domain	Beam
	Shan et al., [279]	Substructure method in combination with RS	Cable stayed suspension bridge
	Moravej et al., [280]	Gaussian Process surrogate model	Large lab-scale box girder bridge
	Su et al., [278]	Gaussian Process surrogate model	Pile-supported wharf structure
	Wu et al., [247]	Kriging Model and Latin Hypercube Sampling method	Truss bridge and an arch bridge
	Mao et al., [249]	Kriging predictor	Cable stayed bridge
	Qin et al., [169]	Kriging model + GA	Complex bridge structures
	Wang et al., [184]	Multi scale model updating using Kriging-meta model	Transmission tower
	Bassoli et al., [166]	Second order surrogate + DEA	Historical masonry structures
	Aruna and Ganguli [21]	Multi fidelity response surfaces	Beam
Damage detection	Fang and Perera [255]	RS (First order response surface models)	Simply supported beam, RC frame, full scale bridge
	Rutherford et al., [283]	RS *meta*-models	Simple linear and nonlinear 5DOF system
	Dey et al., [284]	RS + GA	Thin-walled channel section cantilever beam.
Reliability analysis	Bucher and Bourgund [281]	RS	SDOF system and three bay five story frame
	Das and Zheng [282]	Improved RS	3 bay 12 story frame
	Gaspar et al., [248]	RS	Deck plate element
	Li et al., [258]	Sequential surrogate reliability method	beam, circular pipe structure, speed reducer shaft, cantilever tube, nonlinear oscillator
	Li et al., [250]	t-IRS	Corroded beam structure, Cantilever Tube Structure, 2D Truss structure

achieved. Using Monte Carlo simulation, the surrogate model for the instantaneous response is used to calculate the time-dependent reliability. Rutherford et al., [283] presented the possibility of using response surface metamodels to identify damage in the form of changes in stiffness and damping coefficient. The advantage of the proposed approach is that it requires a relatively small data set and can detect changes in parameters and locations in some nonlinear problems. The effectiveness of the proposed method for damage detection was demonstrated on a simple linear and nonlinear 5DOF system. Dey et al., [284] proposed a method to predict the crack parameters (location and depth) from the measured natural frequencies based on the integration of the finite element response surface method and the genetic algorithm. The verification of the proposed method was performed on a thin-walled channel section cantilever beam. The sum of the previously mentioned research works based on the application of the surrogate-based method to solve finite element model updating problems, indicating the application and the type of structures and models to which it was applied, can be found in Table 10.

Based on the literature review conducted, it can be concluded that replacing the finite element models with an analytically feasible and computationally cheaper surrogate can successfully reduce computation time. In addition to updating the finite element model to determine the unknown structural parameters, surrogate models can be successfully used for damage detection and reliability analysis. They can also handle complex optimization problems, which usually have a computationally expensive objective function. The application of model updating using the response surface method is characterized by difficulties in finding a suitable design, followed by a series of trials and errors with different dimensions and submodel.

7.2. Artificial neural network

Artificial neural networks are an artificial replica of the human brain that attempts to simulate the learning process and tries to implement simplified models that make up the biological neural network. It consists of a series of interconnected simple process elements, units, or nodes whose functionality is based on a biological neuron. The processing power of the network is stored in the strength of the connections between its individual connections, i.e., neurons. Data processing is performed by parallel operation of neural network nodes. The most

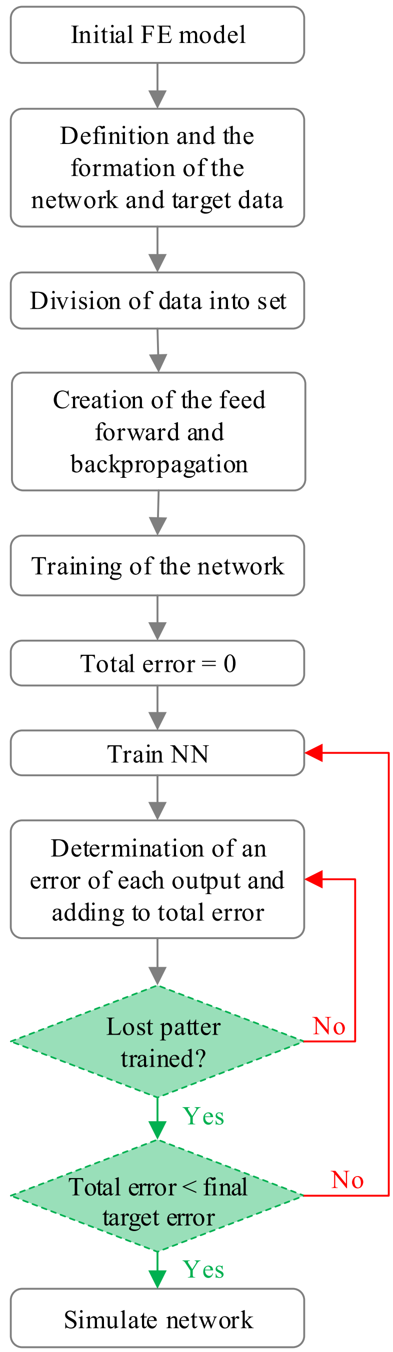

Fig. 11. Neural network a) flowchart b) graphical representation of multi-layer perceptron.

commonly used network topology, the multilayer perceptron (MLP), consists of an input layer, an output layer, and a number of hidden neuron layers. Its parts are interconnected in such a way that the transmission of information is one-way - a neuron sends the information to other neurons without receiving information from them. Such a general configuration of a neural network and a multilayer perceptron is shown in the figure. To ensure and maintain the accuracy of the neural network, it is important to choose the right number of hidden layers and neurons in each layer. The selection of the number of neurons is done by the trial-and-error method [285] or by applying empirical relations [286]. To obtain the output of the neurons, a nonlinear procedure of transforming the weighted sum of the inputs is performed. The back-propagation is determined by the connections between the neurons, the activation function they use, and the learning algorithm that determines the weight adjustment process. During backpropagation, the learning algorithm goes through two phases. In the first phase, the training input pattern is displayed by the input layer of the network. The grid is distributed from layer to layer by the input pattern, and the output layer is generated by the output pattern. If the output pattern deviates from the desired output, an error is calculated and then propagated backward through the network from the output layer to the input layer. According to the error propagation, the value is modified. Based on the above, it can be concluded that learning with multilayer perceptrons (Fig. 11 b)) can be conceptualised as a nonlinear minimization problem that can be solved by applying a gradient-based algorithm. On this basis, the set of the optimal weights are obtained that provide a minimum error value between the neural network outputs and the actual values [286]. The procedure for using artificial neural network in performing finite

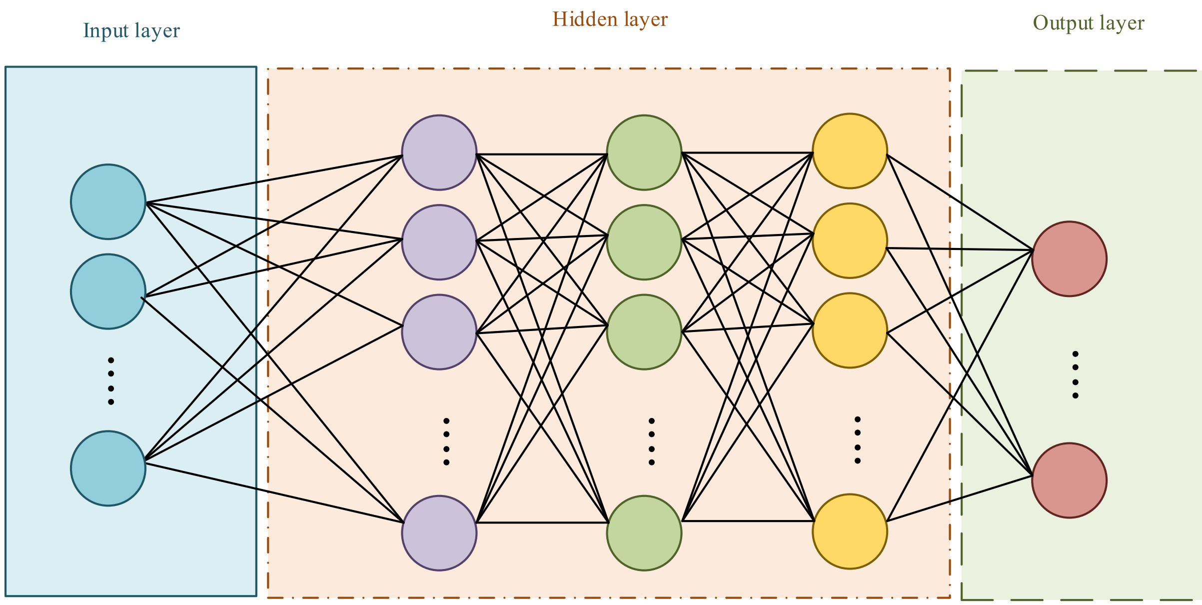

Fig. 11. (*continued*).

element model updating is presented on Fig. 11 a. This method are popular in structural application for model updating and monitoring of frame buildings, bridges and trusses [287]. In this paper some of the studies in which authors have used Neural network for model updating damage detection and improving the computational efficiency of existing methods are singled out.

Kim et al., [108] used the neural network to update the boundary conditions of long span bridge using the static data. After its identification, it was assumed that boundary conditions were fixed, and the stiffness of the structure was updated by dynamic FEMU. In this way, authors improved the parameter identifiability and addressed the limitation of the conventional FEMU method related to using heterogeneous data in FEMU. Sabamehr et al., [176] used Neural network combined with genetic algorithm to find the correlations between the structural frequencies and changes in the sectional properties of the bridge segments. The outputs of these models are compared with outputs obtained by performed the matrix based FEMU method. It was found that the matrix-based method has a better performance in identifying the modal properties. But the main disadvantage of the matrix based FEMU method is its difficult to implementation. Naranjo-Pérez et al., [226] took advantage of individual FEMU methods and approaches, particular harmony search, active set algorithms, machine learning technique and statistical tool in order to propose a new collaborative algorithm and solve the problem of computational efficiency and solving of the additional decision making problem. The validation of the proposed algorithm was performed on the Bormujos footbridge, and it is obtained that the proposed method reduces the simulation time required to perform the optimization of FEMU problem without compromising the accuracy of the solution. The ANN method properties are used to meet the requirement of defining the Pareto optimal front as a convex function. When it comes to uncertainties in the measured data and the developed numerical model, the reliability of the ANN method is questioned. For this reason, several studies [27,288,289] have developed a probabilistic approach for ANN that has led to a promising solution, but due to the complexity in solving practical engineering problems, it is not possible to obtain an unbiased probabilistic distribution of uncertainties. To solve this problem of uncertainties in damage detection using structural dynamic parameters, Padil et al., [261] proposed a non-probabilistic ANN method.

The ANN can be successfully used in order to perform the damage detection of different type of structure using different structural dynamic properties. Yuen [71] dseleescribe the process of damage detection under the ANN using the structural dynamic properties. This process consists of two steps. In first step damage locations are identified using an ANN with damage signatures as the inputs, while in the second phase, the severity of damage forms the first step is estimated by another ANN with structural dynamic properties and the inputs. Both ANNs are design using the Bayesian model class selection method. In their study, Hakim et al., [290] proposed an application of ANN for the prediction of damage severity and location on I-beam structures using natural frequencies and mode shapes. Based on the obtained results, they concluded that ANN can be successfully used to detect single damage and that the combination of natural frequencies and mode shapes is a better option than using them individually.

In addition to the classical structural dynamic parameters (natural frequencies and mode shapes) it has been shown that the damage detection under the ANN algorithm can be performed using the frequency response function [291]. This type of data sets can ensure that the numerical simulations do not need to be performed if the ANN are completely and properly trained. The authors with their study show that several advantages of using FRFs together with the ANN in damage detection over the modal parameters. Most commonly, as several studies show, using the natural frequencies and mode shapes to perform damage detection are limited to quantifying the single damage and large error may be introduced when quantifying multiple damage [263]. Tan et al., [263] proposed in their work a damage detection method for early stage single and multiple damage scenarios on steel beam based on ANN. The proposed method uses the modal strain energy based index to deal with the single damage scenario. On the other hand, the ANN incorporates the modal strain energy based index as the input layer in order to quantify the multiple damage scenarios severities. In order to reduce the dimension of the initial FRF data and transforms it into new damage, Bandara et al., [90] in their study presented the damage detection under the ANN using the frequency response function The artificial neural network is also used to detect different levels of nonlinearities. They showed that the ANN trained with the summation FRF give higher precise damage detection than those ANN trained with individual FRF. This opens the potential of use the proposed method for structural health monitoring application. As the antiresonant frequencies can be identified more easily and accurately than the natural frequencies and mode shapes, Meruane and Mahu [292] developed a real time damage detection under the ANN using the antiresonant frequencies. They verified the proposed method on the steel beam of rectangular cross section.

Table 11
Review of the using artificial neural network for FEMU, damage detection and reliability analysis.

Reported application	Examples of related study	Type of simulated annealing optimization	Structure
FEMU	Kim et al., [108]	ANN	Long span bridge
	Sabamehr et al., [176]	ANN + GA	Bridge segments
	Naranjo-Pérez et al., [226]	HS + active set algorithm + machine learning + statistical tool	Bormujos footbridge
Damage detection	Padil et al., [261]	Non probabilistic ANN	Experimental and numerical model of frame
	Yuen [71]	ANN	Five storey building
	Hakim et al., [290]	ANN	I-beam structures
	Hakim et al [291].	ANN + FRF	Experimental and numerical structure of two-storey steel framed, 38-storey tall building model, cantilevered beam model
	Tan et al., [263]	ANN + MSE	Steel beam
	Bandara et al., [90]	ANN + FRF	Three-story bookshelf structure
	Meruane and Mahu [292]	ANN + antiresonant frequencies	Eight DOF spring mass system, steel beam, with a rectangular cross-section

Based on the literature review of finite element model updating and closely related applications using the ANN method (Table 11), it can be concluded that this method is successfully applied. Moreover, it is also successfully used to solve the problem of uncertainties in the measured data and the developed numerical model. Due to the ability to model the nonlinear relationship between the structural dynamic properties and the location and intensity of damage, ANN has also proven to be a very effective method for damage detection. In addition to the classical structural dynamic properties - natural frequencies and mode shapes - it can be successfully used for not only damage detection but also structural health monitoring using the frequency response function. Some of the advantages of ANN over some conventional traditional algorithms are very good estimation of nonlinear relationship, ability to work with ambiguous or deficient data and detect patterns, robustness to data errors, working with many variables or parameters, formulation of knowledge based on experience, etc. The main problem of ANN is the large amount of training set required to properly train the network, based on which the process of model updating, and closely related processes can be performed as accurately as possible.

8. Bayesian finite element model updating

The basic principle of the Bayesian method is that uncertainty parameters, modelling error, and measurement error are modelled as design variables. The probability density function for the measurement error and modelling error is defined and parameterized with parameters whose values account for both errors. These parameters are added to the parameters of the numerical model of the structure by forming a set of general parameters. This is equivalent to adding the probabilistic model classes to the design model class parameterized with parameters that include model parameters, measurement parameters, and error parameters. This procedure is commonly referred to as stochastic embedding [3].

As mentioned earlier, the probability density function reflects the situation where the model class, $\mathscr{M}_m$, and the experimentally obtained data set are known. Most often, it is selected based on engineering judgement and determined independently of the measurement results. Due to the significant influence of the PDF on the results, the problem of arbitrariness and subjectivity arises. When dealing with real construc-

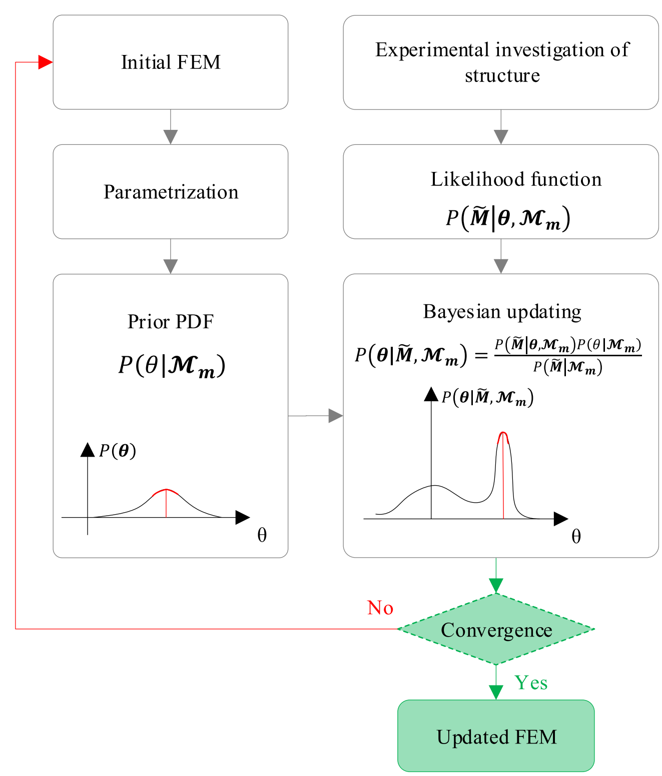

Fig. 12. Finite element model updating under the Bayesian method.

tions and real systems, the calculation of joint and marginal PDFs involves a large number of parameters. This leads to high-dimensional integrals for which approximate measures or sampling methods, such as the Markov Chain Monte Carlo method, are used to solve. If a conjugate prior is used, the posterior probability density function can be determined numerically. If the number of parameters is limited, the posterior PDF can be determined analytically. After the posterior PDF is calculated, estimated, or approximated, it can provide information on how much the uncertainty of the parameters decreases relative to the observed data and the available prior information. The posterior probability density function can be approximated in a number of ways, including the Gaussian distribution, asymptotic approximations, or sampling techniques. If both the prior PDF and the likelihood function are Gaussian, the posterior PDF will also have a Gaussian distribution [127]. Asymptotic approximations are used when a large amount of data is available. The most popular Markov chain Monte Carlo method is used to sample the posterior PDF and improve the convergence speed. Fig. 12 shows a flowchart of finite element model updating under the Bayesian approach, while in the following, the applications of Bayesian method in solving various problems of finite element model updating are discussed.

Li et al., [293] studied the Runyan suspension bridge in detail based on a series of field vibration data. They used a probabilistic FEMU to update the stiffness of the pile foundation. In order to minimize the error function, where both natural frequencies and mode shapes were selected as target responses, they performed the updating procedure using the Bayesian algorithm based updating software FEMtools. Altunişik et al., [40] performed the Bayesian model updating on the historical timber mosque based on vibration testing to minimize the differences between numerical and experimental results. FEMU was used to perform the linear time history analysis and structural behavior evaluation before and after model updating. Based on the results, they concluded that model updating is very effective in reflecting the actual behavior of the structure and obtaining its response. In another study [42], the same author also performed Bayesian finite element model updating using ambient vibration, but in this case for a historic masonry structure to reduce the differences between the actual and predicted structural behavior. The study concluded that the locally updated model appears to be better able to provide accurate predictions for structural behavior evaluation and that the model can be used as a starting point for SHM. Ponsi et al., [294] used Bayesian updating of the finite element model to perform the calibration of the complex FE model of a historic masonry fortress damaged by a seismic event. Fujita and Takewaki [295] proposed a statistical update of the finite element model to determine the story stiffness of a shear bending model. In the proposed method, the probability distribution of floor rotation angle in the lowest mode is obtained for the identified shear bending model. By providing additional measurement data on the floor rotation angle, a conditional probability problem is applied. In order to determine the stiffness of the linkage between two parts of the coupled building, Hu and Yang [296] studied the coupled building with two linked shear buildings and updated the numerical model using also Markov Chain Monte Carlo based Bayesian model updating. Lam et al., [297] performed Bayesian model updating of a 20-story office building by performing a full-scale vibration test to determine the distribution of inter-story stiffness. This is made possible by applying Bayesian model updating using the Markov Chain Monte Carlo Bayesian model updating to explicitly address the uncertainties. Asadollahi et al., [63] proposed a Bayesian inference method using the Transitional Markov Chain Monte Carlo (TMCMC) algorithm to marginalize the prediction error precision. The proposed method was applied to update a long-span cable-stayed bridge using long-term monitoring data collected from a wireless sensor network (WSN). The proposed method was compared with constant error precisions [298] and updating error precisions [299]. The comparison showed that the prediction error precisions with marginalized treatment performed best in terms of more accurate inference of model parameters and quantification of posterior uncertainty MU. In this way, more reliable predictions of model properties were achieved. Argyris et al., [300] used the TMCM to performed the high fidelity finite element model updating of the highway bridge. In their study, they proposed a likelihood formulation that included the mode shapes based on the probabilistic treatment of the MAC value. Sun and Büyüköztürk [301] proposed a new probabilistic model updating using incomplete modal data. To solve the modal matching problem in model updating, a new strategy for Bayesian model updating was proposed using a model reduction technique and MCMC with adaptive random walks. The effectiveness and capabilities of the proposed method were tested and confirmed on a nine-story building with synthetic measurements The application of the proposed approach does not require any adjustment of mode shapes, while it is realized by model reduction. Yuen and Ortiz [287] proposed a novel nonparametric general Bayesian regression method with multiple resolutions for MU using modal data. The FEMU problem is posed as a nonlinear regression problem from the modal data to the structural parameters. It does not require an explicit functional form but uses the input–output data to adaptively model the relationship. The proposed method was tested on a 20-storey shear building and a 3D truss. The results show that the method is very simple, effective, and computationally inexpensive. To solve the problem of updating the model of a nonlinear dynamic system with nonclassical damping, Cheung and Bansal [302] proposed a new Gibbs-based approach to Bayesian model updating for linear dynamic systems. The proposed approach is based on the incomplete modal data including natural frequencies, mode shapes, and damping ratios. The results from the performed numerical examples showed that the proposed approach is useful only for globally identifiable and unidentifiable contribution sums of the corresponding mass and stiffness matrices from each prescribed substructure. In addition to performing FEMU with the aim of obtaining structural parameters and properties, the Bayesian method and its derivation are also used for damage detection. In order to estimate the joint posterior probability distribution of the updating parameters of a damaged four-story masonry reinforced concrete structure, Akhlaghi et al., [303] performed Bayesian model updating using ambient vibration data. Based on the conducted research, it was shown that the Bayesian finite element model updating method can detect and identify damage. Instead of a FEMU optimization problem that uses an objective function or modal fitting measures, Kernicky et al., [304] proposed an approach that solves the entire feasible parameter space while satisfying the constraints. They extended the application of nonlinear constraint satisfaction with interval arithmetic to larger system models with multiple degrees of freedom based on the prior computation of a full set of feasible solutions. In this way, they enabled the identification of uncertain stiffness parameters and the exact identification of damage for all cases of severity. Moravej et al., [305] used for the first time the modular Bayesian approach to update the initial laboratory-scale numerical model of a concrete box for two states - an undamaged and a damaged state based on the results of an experimental modal analysis. In this study, the FE is replaced by a Gaussian process model as a metamodel. The main advantage of using the Gaussian process model is that it takes into account the main sources of uncertainty. Based on the previously stated, the results of the performed model updating performed are realistic. The results of the analysis show that the reduction of the stiffness in the damaged state is significant, and that this reduction corresponds to the cracks observed on the structure. In damage detection, the main problem is the amount of modal data, which is often insufficient to perform a model updating for damage detection, and its insensitivity to the location of the damage. Therefore, Hou et al., [306] proposed a method for damage detection that solves the above problem and is based on additional virtual masses and Bayesian theory. The use of additional virtual masses was used to obtain a larger number of virtual structures and a lot of modal and statistical information. Bayesian theory was used to obtain the posterior probability density function to determine the damage factor. Astroza et al., [307] provided a tool for post-disaster damage identification and SHM using a Bayesian inference method and dynamic input–output data recorded during an earthquake. The effectiveness of the proposed method was demonstrated on a RC frame building subjected to bi-directional horizontal seismic excitation. The results showed that the updated FE model can be used to reconstruct the unmeasured responses from the local to the global level and to estimate the type and extent of damage to the entire structure. To include damping measurements in the update, Das and Debnath [126] proposed a Bayesian finite element model updating based on a maximum a posteriori using the incomplete complex natural frequencies and mode shapes. In their study, in addition to the mass and stiffness, the damping parameters were also updated in the form of classical damping and non-classical viscous damping. Compared with Gibbs sampling technique and sensitivity method, the proposed method showed better performance in terms of computation time, convergence rate and number of iterations. Using Bayesian FEMU in combination with input–output data obtained during small, medium, and high amplitude dynamic excitation, Ebrahimian et al., [308] proposed a framework for SHM and damage detection. The possibility of applying the proposed method in SHM and damage detection after an earthquake was ensured by extracting data on different manifestations of damage in parts of the updated structural model. The proposed method can also be used to locate, classify, and quantify damage at local and global levels of the structure. In addition to the data obtained by SHM and performing static and dynamic tests, the use of longitudinal guided wave signals was also noted. Thus, using guided wave signals, Ng [309] proposed a Bayesian model updating approach for quantifying damage in beam-like structures. The proposed method is applicable not only to laminar damage, but also to other types of damage or more complex engineered structures. This is possible by modifying the embedded spectral finite element model in the Bayesian framework. The proposed approach can accurately identify the damage even when the extent of the damage is small. In addition to the previous examples of studies where the Bayesian approach and its derivation have been used for finite element model updating and damage detection, it can also be used for model class selection. Chiachío et al., [65] used the Bayesian approach to select the most probable model class for fatigue damage prediction in composite materials. On the other hand, Goller et al., [111] in his study proposed a method to select the weights of the probability density functions for the case where the likelihood function is formulated as the product of two probability density functions related to different data sets. The proposed method is based on the performing the Bayesian model updating at the class level and was verified using simulated data for a 2 DOF system and using experimental

Table 12
Review of the using probabilistic method for FEMU, damage detection and model class selection.

Reported application	Examples of related study	Type of probabilistic method	Type of structure
FEMU	Li et al., [293]	Bayesian method	Runyan suspension bridge piles
	Altunişik et al., [40]		Historical timber mosque
	Altunişik et al., [42],		Historical masonry structure
	Ponsi et al., [294]		Two floor frame, San Felice sul Panaro fortress
	Fujita and Takewaki [295]	Subspace method	SB model, 10 story plane building frame
	Hu and Yang [296]	MCMC	Coupled buildings
	Lam et al., [297]		20-storey office building
	Asadollahi et al., [63]	TMCMC	Cable-stayed bridges
	Argyris et al., [300]	TMCMC of high fidelity finite element model	Ravine highway bridge
	Sun and Büyüköztürk [301]	Model reduction techniques and MCMC with adaptive random walks	Nine-story shear type building
	Yuen and Ortiz [287]	Multiresolution Bayesian nonparametric general regression method	20-storey shear building, and 3D truss.
	Cheung and Bansal [302]	Gibbs based approach for Bayesian model updating	4-DOF mechanical system, 120-DOF. Frame structure
Damage detection	Akhlaghi et al., [303]	MCMC	four-story masonry infilled reinforced concrete structure
	Kernicky et al., [304]	Constraint satisfaction with interval arithmetic	Planar truss
	Moravej et al., [305]	Modular Bayesian method	Lab-scaled concrete box girder bridge
	Hou et al., [306]	Bayesian theory in combination with additional virtual masses	3 story frame structure
	Astroza et al., [307]	Bayesian inference methods	RC frame building
	Das and Debnath [126]	Bayesian method based on MAP	Spring-mass-damper system ASCE benchmark structure
	Ebrahimian et al., [308]	Batch Bayesian estimation method integrated with mechanics based nonlinear FEM	Cantilever steel column 2D moment resisting steel frame
	Ng [309]	Bayesian method	Beam like structure
Model class selection	Chiachío et al., [65]	Bayesian method	Composites element
	Goller et al., [111]		2 DOF system using simulated data and 2 DOF planar shear building using experimental data
	Simeon et al., [298]		analytical example of simple linear regression, beam

data for a 2 DOF planar shear building. Simeon et al., [298] studied the effect of prediction error correlation on the results of a conductive model updating using the Bayesian method and also studied the effect of selecting a prediction error correlation structure. Based on a fair study of the analytical example of simple linear regression and the example of reinforced concrete beams, they concluded that the Bayesian class selection model can be applied in solving the problem of selecting the appropriate prediction error correlation structure and that more realistic modelling and updating results can be provided. The sum of the examples of studies in which the probabilistic approach and its derivate used to perform finite element model updating, damage detection is shown in the following table (Table 12).

In addition to those examples, in table the three examples of study in which the Bayesian approach was used to perform the model class selection concerned with the mathematical hypothesis of the ability of the models to predict measured quantities is also given. Based on the literature review conducted on probabilistic (Bayesian) model updating, it can be concluded that it can be successfully used for solving various types of parameter estimation and model selection problems, more so than what was found in the literature cited in this paper. But, in addition to their successfully application those methods can often found to be complex, time consuming and computationally costly process. Those most often limit their application on large civil engineering application.

9. Fuzzy finite element model updating

Comparing with the Bayesian probabilistic finite element model updating method where the probability density function is defined, in fuzzy finite element model updating method, instead the probability density function, the fuzzy membership function is defined in order to define the uncertainty related to the measured outputs. In addition, at different level of the membership function, the interval finite element model updating is applied. Fuzzy method is based on three processes: the fuzzification process, the inference process, and the defuzzification process. In the fuzzification process, the degree of membership of variables is calculated based on a certain membership function. All variables with possible values (sets) are within certain rules. In the process of inference, the variables are transformed for a particular situation. In the process of defuzzification, the values of variables are transformed into numerical values. In the process of updating the finite element model using the fuzzy method, the process begins with the initial estimation of the membership functions of an input parameter. Then, in the next step, the fuzzy membership function of the outputs is used. At different levels of the membership function, interval model updating is performed. In this way, the initial fuzzy membership of the input parameters is updated (Fig. 13).

The following are examples of research where updating of fuzzy finite element model updating has been performed to improve numerical models of various types of structures, as well as simple and complex numerical and laboratory models based on various data sets obtained through experimental testing.

Khodaparast et al., [142] proposed a method for calculating the measured fuzzy membership function for experimental data and a method for extracting membership functions for histograms adopted to the data. The measured fuzzy functions were used to update a fuzzy model of a 3-DOF mass spring system and applied to a DLR AIRMOD structure. To consider the uncertainty of the measured modal data by using only a single modal test, Liu and Duan [144] proposed the fuzzy FEMU method to update the FE model of a practical bridge. Compared with the stochastic FEMU methods, it was shown that the proposed method does not require measured structural dynamic parameters obtained from couples of structural dynamic tests. On the other hand, the proposed method requires more computational effort compared to the regular FEMU methods. In general, the fuzzy method is very computationally cost for dealing with the effects of uncertainties on the system behaviour. This problem is overcome by using metamodels (surrogate models), surface methods, neural networks and Kriging predictors [144]. Erdogan and Bakir [310] used fuzzy FEMU to represent the uncertainties in the dynamic structural parameters caused by measurement noise. To minimise the objective function to obtain the membership function of the uncertain parameters, the authors used a genetic algorithm. In another work, Erdogan et al., [106] used a fuzzy set-based approach to quantify the uncertainty to investigate the uncertainty variations in the model results of the two-span benchmark structure. To perform the more successful FEM calculations, they used the Gaussian process (GP) model as a surrogate model. Bulkaibet et al., [145] used fuzzy FEMU based on metaheuristic optimization algorithms to quantify the uncertainty associated with the modal parameters of a 5-DOF mass-spring system. To simplify the computational process by transforming the fuzzy calculations into a series of interval calculations, they used the α-cut method. They compared the proposed method with the PSO and Bayesian algorithm. The comparison showed that the ant colony optimization (ACO) algorithm was able to achieve very good results with the few ants. Sun et al., [181] proposed an innovative model updating techniques for the evaluation of a cable-stayed bridge. In the first phase, a sensitivity analysis is used to select the updated parameters. In the second phase, a fuzzy outranking method is applied to evaluate the non-inferior solutions from the first phase. Moreover, the best compromise solution for updating the parameters is determined. Dominik and Iwaniec [311] proposed a new method for damage detection to electric pylon using the fuzzy logic method applied to the comparison of natural frequencies. Mojtahedi et al., [312] proposed a Fuzzy- Krill Herd algorithm to perform SHM and model updating of a fixed jacket platform to perform damage detection. The proposed method is based on the Krill Herd analogy, which consists of three different actions: (1) motion introduced by other krill, (2) foraging activity and (3) random diffusion. Based on these three steps, the current position of each krill swarm can be determined. The objective function in the proposed methodology represents the smallest distance between the position of the food and the highest density of the herd. In addition, the authors performed a comparison of the proposed method with the traditional fuzzy genetic method. The comparison has shown that the fuzzy genetic algorithm is insignificantly better compared to Fuzzy Krill Herd algorithm, but both algorithms do not differ significantly. It has also been shown that the FKH algorithm requires a lower number of iterations in some cases and is more suitable for fuzzy coupling.

Previous research in the non-probabilistic finite element model updating, i.e. fuzzy logic (Table 13) showed it successful application which can be used in order to quantify different type of uncertainties and their effect. In addition, its successful application for model updating and damage detection are also reported. This method is valuable source for modelling the uncertainties when, in addition to the interval bounds, the uncertain quantities are also available. The power of this method lies in the gradual description of membership interpret depending on the specific application. The use of fuzzy logic is a particular advantage in decision-making processes where description by algorithms is extremely difficult and criteria are multiplied.

10. Regularization method

The update of the finite element model and the related problem can be considered as an optimization problem aimed at finding the parameter vector that minimizes the difference between the measured and the calculated responses. The identified equation is typically an ill-conditioned inverse problem. To improve this problem and ensure that the obtained results have high robustness, the regularization method - Tikhonov (ℓ2-norm) and sparse (ℓ1-norm) regularization is used. Due to the possibility of obtaining a sparse solution, the sparse regularization method is most often used for structural damage detection. In general, regularization is a process in which an additional penalty function is introduced to solve an ill-posed problem or prevent overfitting of the model [315]. Essentially, this ensures that the finite element model does not overfit the measured data, to the expense of the

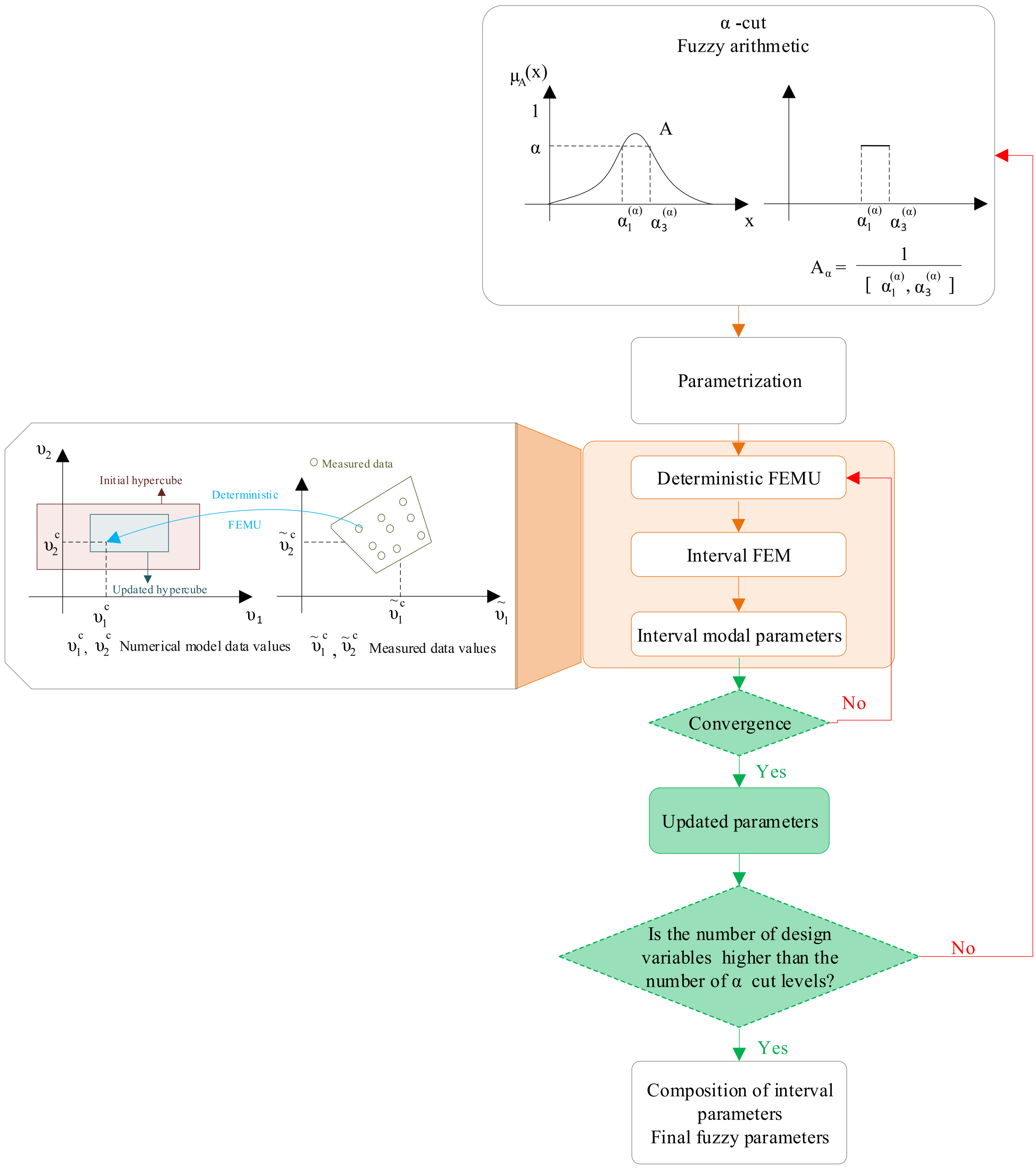

Fig. 13. Finite element model updating under the fuzzy method flowchart.

physics in the finite element model. Following the original idea, regularization extends the objective function to new conditions that depend on updating the parameters rather than the measured responses. This optimizes the search domain that is assumed to belong to it. The problem that the regularization method tries to solve has the following form (Eq (17)):

$$\min_{\{\theta\}\in R^N} \|r(\theta)\|_2^2 \tag{18}$$

where θ are the indices of the structural parameters, r(θ) is the residual or difference between the predicted and the actual structural behaviour and can usually be defined using the dynamic structural parameters. The residuals corresponding to small changes in a structural parameter can be linearly related to small changes in Δ θ as follows:

$$r = S\Delta\theta + \varepsilon \tag{19}$$

where ε is error vector which represent the effect of measurement and numerical error, while S is a sensitivity matrix defined as:

$$S_{ij} = \frac{\partial r_i}{\partial \theta_j} \tag{20}$$

The regularization term is added in the objective function as follows:

$$\min_{\{\theta\}\in R^N} \|M(\theta) - \widetilde{M}\|_2^2 + \beta\|\{\theta\}\|_2^2 \tag{21}$$

where $\|M(\theta) - \widetilde{M}\|_2^2$ is the residual normal which proposed the differences between the structural behaviour predicted by numerical model, $M(\theta)$, and its actual behaviour $\widetilde{M}$, β is the regularization parameters while the $\|\{\theta\}\|_2^2$ is the regularization term or norm solution. The proposed equation is an example of the l2 regularization, while the l1 norm regularization is defined as:

$$\min_{\{\theta\}\in R^N} \|M(\theta) - \widetilde{M}\|_2^2 + \beta\|\{\theta\}\|_1 \tag{22}$$

Table 13
Review of the using non-probabilistic method for FEMU, quantification of the uncertainties, and damage detection.

Examples of related study	Type of non -probability method	Reported application	Structure
Khodaparast et al., [142]	Fuzzy finite model updating	Finite element model updating	3-DOF mass spring system; DLR AIRMOD structure
Liu and Duan [144]		Consideration the effect of the measured uncertainty of modal parameters on the updated model.	Continuous prestressed concrete bridge
Erdogan and Bakir [310]		Quantification of uncertainties of the measured uncertainties of differently type of measured parameters	Reinforced frame structure
Bulkaibet et al., [145]			5-DOF mass-spring system
Moens and Vandepitte [313,314]			Plate with uncertain boundary conditions; Garteur benchmark problem; solar panel; COROT baffle cover
Erdogan et al., [106]	Fuzzy set-based uncertainty quantification approach	Investigation the effect of uncertainties on the predicted response of structures	Two-span benchmark structure
Sun et al., [181]	Two-phase FEMU method based on sensitivity analysis and fuzzy outranking method	Assess the mechanical state of structure	Cable-stayed bridge
Dominik and Iwaniec [311]	Fuzzy logic method based on the natural frequency comparison	Finite element model updating for damage detection	Electric pylon
Mojtahedi et al., [312]	Fuzzy Krill Herd algorithm	Finite element model updating for damage detection	Offshore jacket platforms

The most important thing when performing finite element model updating or damage detection using the regularization method is to choose the right regularization parameter β. The best way to define it correctly is to find a suitable compromise between data fidelity and sparsity solutions. In the Tikhonov regularization, the L-curve criterion is used, which satisfies the small norm of the solution and the small norm of the residual. For the $\ell 1$-regularization problem, the curves of the residual norms versus the solution norm do not have the form L and are therefore plotted in a linear scale, unlike the $\ell 2$-norm where they are plotted in the log–log scale. Since the $\ell 1$-norm does not have a smooth solution and the curvature of the L-curve is not identified, it cannot be expressed explicitly [316]. The main differences between the Tikhonov regularization and the sparse regularization are based on linearity, convergence to zero, regularization path, and sparsity [317]. Moreover, the l1 minimization requires more computational effort when comparing the computational efficiency and the obtained solution, and there is no closed-form solution. The Tikhonov regularization ($\ell 2$-norm) is a common approach to the insufficiency problem and provides an acceptable and smooth solution. It is somewhat more widely used due to its computational efficiency and ease of implementation. However, since the $\ell 2$-norm promotes smoothing of the solution, this solution is sometimes over-smoothed, especially when the number of sensors is limited. On the other hand, the solution of the inverse problem in the context of damage detection usually has sparse properties, since typically only a small number of structural components are damaged compared to the whole structure. In the following, an example is given where the regularization method is used to solve the problem of updating the finite element model in the context of damage detection and determining the optimal value of the structural parameters.

In the context of the finite element model, the regularization method has been successfully used [318]. To solve ill-posed invers problem of finite element model updating of a bridge with two continuous spans, Rezaiee-Pajand et al., [99] proposed a new iterative and hybrid regularization method based on the Krylov subspace theory [319] and the bidiagonalization process. The proposed method was compared with the least square minimum residual method (LSMR), Reg-LSMR (further development of the LS problem by adding regularization parameters), and hybrid LSMR (LSMR combined with the Tikhonov regularization method). The comparison showed that the proposed sensitivity-based strategy and the regularized solution method are influential and successful in model updating under incomplete noisy data. Luo and Ling [232] proposed a particle swarm optimization based sparse regularization approach for structural damage detection. Classical first-order sensitivity analysis and $\ell 1/2$-norm regularization were introduced to define an objective function solved by PSO. The proposed method was verified on a 10-element cantilever beam and was shown to be capable of detecting locations and accurately quantifying the extent of damage. Hernandez [317] provided a new theoretical basis for the identification of localized damage in structures in terms of stiffness reduction using incomplete modal information. Based on the study conducted, it was concluded that the l1-based sensitivity approach can accurately identify damage using noisy data. Zhou et al., [320] proposed a new l1 regularization approach to identify damage using data from the first few frequencies. The proposed method is mainly based on the sparse vector theory, since a sparse vector can be successfully recovered with a small number of measured data. The advantage of the proposed method is that the first few natural frequencies can be measured more accurately than the mode shapes. In addition, the authors conducted a study on the influence of damage severity, number of damages, number of measurement data, and noise on the damage detection results and came to the following conclusions. The number of measurement data, measurement noise, and damage severity affect the accuracy of damage detection. More severe damage, less measurement noise, and more measurement data generally improve damage detection. If the frequency changes caused by the damage are significantly higher than the frequency changes caused by the noise, the damage can be detected correctly. Hou et al., [191] proposed a l1 regularization-based technique for model updating by utilizing the sparsity of structural damage and developing a strategy to select the regularization parameters for the l1 regularization problem. The effectiveness of the proposed method was demonstrated on a truss structure and a three-story steel frame. Hou et al., [321] used the l1-norm regularized damage detection for a cantilever beam and a three-story frame using optimal sensor data. The optimal sensor placement was determined using the OSP method based on GA. To provide the best conditioned model updating scenario for the target structure, Garcia-Palencia et al., [322] proposed a regularization technique and frequency point selection protocol for updating the university of central Florida benchmark structure. In the initial phase, the model updating was performed by changing the stiffness, mass, and damping parameters to obtain the initial undamaged numerical model. Then, the process was repeated in the damaged state, using the updated parameters of the initial model and the subsequent damaged state for damage detection. The updating of the model was also performed in two steps: In the first step, the stiffness was calibrated and in the second step, the damping was calibrated. According to the proposed approach, the frequency points for the first step of the procedure should be outside the resonance, anti-resonance and noisy regions in the experimentally obtained FRFs, while for the second step, when the identification of the damping is performed, the frequency points in the resonance are needed because the damping causes the significant changes in the response. Zahn and Xu [323] proposed an alternative method based on the sparse regularization (ℓ1-norm regularization) to solve the problem related to the ill-posedness in response sensitivity based damage detection. The effectiveness and superiority of the proposed method was tested on an overhanging beam. Shahbaznia et al., used sensitivity analysis and Tikhonov regularization methods to solve the inverse problem of model update and damage detection in the time domain of railway bridge without knowledge of the moving load. Pan and Yu [324] proposed a sparse regularization based method for damage detection in a beam bridge using only structural responses caused by an unknown moving force. To improve the ill-conditioned problem, Lp-norm sparse regularization was used in the proposed method. Ding et al., [325] proposed a new non-probabilistic method based on Hybrid C-Jaya-TSA. In the proposed method, the authors used sparse regularization technique to solve the problem of limited number of measured data and performed damage detection on TV tower, truss model and cantilever beam. Entezami et al., [160] proposed a new regularization method, Regularized Least Squares Minimal Residual (RLSMR), to solve the problem of damage detection based on sensitivity using incomplete data contaminated by noise. The proposed approach is based on the Krylova subspace and uses bidiagonalization and iterative algorithms to solve systems of linear equations. The comparison of the proposed approach with Tikhon's regularization method has shown that the proposed approach gives better results in damage detection. Hua et al [326] addressed the determination of regularization parameters in the implementation of the Tikhonov regularization technique in finite element model updating. They formulated an adaptive strategy that allows varying the value of regularization parameters in different iteration steps. The optimal value of the parameters is determined based on the minimum product criterion. The comparison of the proposed method with the L-curve method and the generalized cross-validation of the truss bridge has shown that the proposed strategy is more effective than the method that uses a constant value for the regularization parameters.

Based on the literature review and previous studies (Table 14), it can be seen that model updating can be successfully applied using the regularization method. As has been previously reported in the literature, the regularization method is used particularly successfully when the data is contaminated with noise. For these datasets, the main problem is that they are sensitive to small fluctuations in outputs that lead to unreasonably large fluctuations in the values of the damage and design variable values during model updating. Another problem that can also be successfully solved by applying regularization methods is the problem of insufficient definition of a system of equations whose solution leads to an infinite number of solutions. A number of recent studies have shown that regularization methods can be successfully applied even in cases where the amount of vibration data is limited. They can provide a satisfactory solution for damage detection requiring a small number of measurement points. In addition, it was shown that the regularization method requires a lot of computational effort which is not justified by improving accuracy.

11. Conclusion

To better develop the numerical model of actual structural behaviour and make its predictions as credible as possible, it is increasingly common to combine numerical modelling with the results of experimental investigation of the structure. This article provides an overview of the process of updating finite element models based on the results of experimental investigations and the most used methods. Several relevant conclusions can be drawn from the literature review and based on the wrote paper:

1. Considering the actual behavior of the structure and the parameters that most credibly represent it, one can conclude that the numerical model of the structure can be improved based on the experimentally determined dynamic properties - natural frequencies and mode shapes. The change in structural dynamic parameters is most associated in the changes of structural stiffness in form of the damage. This prove that these parameters are in fact the first indicators of the occurrence of a damage on the structure.
2. In addition to the use of the structural dynamic properties, the static data sets can also be used for updating the finite element model, especially for numerical modelling of complex structures.

Table 14
Review of the using regularization method for FEMU and damage detection.

Reported application	Examples of related study	Type of regularization method	Structure
FEMU	Rezaiee-Pajand et al., [99]	Krylov subspace method	Two story concrete frame, two span continuous steel truss
Damage detection	Luo and Ling [232]	PSO based sparse regularization	Cantilever beam
	Hernandez [317]	L1 norm regularization	Shear-beam, plate in bending
	Zhou et al., [320]		Cantilever beam
	Hou et al., [191]		Cantilever beam, three story frame
	Hou et al., [321]		Planer truss, three story steel frame
	Garcia-Palencia et al., [322]	Tikhonov regularization	USF-Benchmark Structure
	Zahn and Xu [323]	Tikhonov regularization and sparse regularization	Planer truss, Overhang beam
	Shahbaznia et al., [31]	Tikhonov regularization using L-curve	Railway Bridges
	Pan and Yu [324]	Sparse regularization	Two span continuous bridge
	Ding et al., [325]		TV Tower, truss model, cantilever beam structure
	Entezami et al., [160]	Regularized Least Squares Minimal Residual	Truss bridge
Output error based FEMU	Hua et al., [326]	Tikhonov regularization in conjunction with MPC	Truss bridge

The main problem with static measurements is the placement of the sensors and the errors they are subject to (size, position, orientation, thermal expansion, reading technique, or measurement accuracy). While, in combination with the dynamic data sets and with the proper definition of the weighted factor values, their combination can give a very accurate and reliable numerical model of structure.

3. Selecting the appropriate design variables have a significant influence on reducing errors and simplifying the finite element model. The best way to perform model parameterization is sensitivity analysis, which results in suppressing the problem of inadequacy. Based on this, the parameters that do not affect the output results are excluded from the model updating process. It is very important that the selected design variables represent the real structural behavior as well as possible.
4. The finite element model updating methods currently represented in literature are divided in two main groups: direct (non-iterative) and indirect (iterative) methods. Direct methods straightly update the elements of the stiffness or mass matrix in one step and provides dynamic parameters corresponding to those obtained by experimental tests. The study addresses the limitation of the direct FEMU, although it reproduces the structural dynamic parameters without the guarantee that it accurately reproduces the actual values of the physical parameters of the structure
5. The most studies have relied on iterative maximum likelihood method that is performed using the single or multi objective approach. The single objective approach obtains only a single solution, which is a subset of the set of possible solutions. Moreover, in this approach, the effect of the weighted factors on the defined single objective function had to be analysed. In contrast, in the multi-objective approach, one obtains a set of possible solutions, called the Pareto optimal front. Determination of the Pareto optimal front is very computationally intensive and time consuming. Moreover, the best possible solution can be found by solving the decision-making problem (best balanced solution), which allows updating the numerical model to better reflect the actual structural behavior. Finally, compared to the single-objective approach, the multi-objective approach shows better performance in solving the finite element model updating problem.
6. Iterative sensitivity-based methods allow many updating parameters and measured outputs and many of them require a high computational effort. Moreover, the sensitivity equation is generally a nonlinear problem linking the input parameters of the numerical model and its output, so an iterative procedure must be performed. The literature identifies the convergence problem in determination of the updating parameters values.
7. The iterative FEMU methods currently represented in the literature are divided into stochastic (Bayesian) and deterministic (maximum likelihood). The existing research has highlighted the advantage of the stochastic method, which provides the overall probability of the distribution of the physical parameters under consideration, while the main drawback is the time required to perform FEMU of complex structures. Compared to stochastic methods, deterministic method provides the point of estimation of the expected value. On the other hand, the reduced time required to calculate the FEMU of a complex structure has led to the deterministic method being widely used for practical applications in civil engineering. Therefore, several maximum likelihood methods optimization algorithms have been proposed to deal with the FEMU problem. However, the Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) are the most commonly used, since their code is usually integrated into numerical computation software.
8. The paper reviews the Harmony Search (HS) and Simulated Annealing (SA) optimization algorithm, where the previous studies have shown that the harmony search algorithm is the most efficient without compromising accuracy compared to GA and PSO. In addition to using optimization algorithms as standalone algorithms, their drawbacks are minimised by combining them with other methods in order to improve the computational efficiency.
9. Surrogate-based finite element model updating shows good performance by replacing the finite element model with a mathematical model that is analytically more practical and computationally more effective. In order to improve the computational efficiency in FEMU, the literature discussed the importance of sampling in creating effective surrogate models. The main problem is to find a suitable design, followed by a series of trials and errors with different dimensions and submodels.
10. The capabilities of the Artificial Neural Network method are also successfully used for solving various types of FEMU problems, especially when consider nonlinear relationship between the damage detection (location and severity) and the measured structural dynamic properties. Moreover, this method can also be used for Structural Health Monitoring by using the Frequency Response Function instead of the traditionally used structural dynamic parameters (natural frequencies and mode shapes). The main problem with this method is the data sets needed to properly train the network.
11. The Fuzzy approach is a kind of stochastic approach in which the updating of model is represented as a non-probabilistic problem, in contrast to the Bayesian method (probabilistic problem). Therefore, the results obtained with these two methods could only be compared qualitatively. The fuzzy finite element model updating can provide results that are mostly easy and intuitive to interpret, and their further processing can provide information about the resulting uncertainties. In addition, the fuzzy approach does not take into account the dependence between the model parameters and/or the experimental data, while the Bayesian approach automatically incorporates the interaction between the two data sets.
12. The regularization method is used to solve the problem of ill-conditioning and overfitting when the number of available measurement data is limited. The most important thing in this method is the proper definition of the regularization parameter. This method is especially successfully used in damage detection. On the other hand, when the number of sensors used for the experimental study is limited, the solution obtained by the regularization method can sometimes be over-smoothed. Also, the problem presents a situation where a small number of structural components are damaged compared to the whole structure which results in sparse properties of the solution. Although it can be successfully applied in improving the accuracy of the solution, it did not justify the required computational effort.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

This research was funded by the European Union through the European Regional Development Fund's Competitiveness and Cohesion Operational Program, grant number KK.01.1.1.04.0041, project "Autonomous System for Assessment and Prediction of infrastructure integrity (ASAP).

References

[1] Kaveh A, editor. Computational Structural Analysis and Finite Element Methods. Cham: Springer International Publishing; 2014.
[2] Bathe KJ. Finite element procedures. 2nd editio,. Massachusetts; USA: Prentice Hall, Pearson Education; 2014.
[3] Simoen E, De Roeck G, Lombaert G. Dealing with uncertainty in model updating for damage assessment: A review. Mech Syst Signal Process 2015;56:123–49. https://doi.org/10.1016/j.ymssp.2014.11.001.
[4] Ye X, Chen B. Model updating and variability analysis of modal parameters for super high-rise structure. Concurr Comput 2019;31:1–11. https://doi.org/10.1002/cpe.4712.
[5] L. He E. Reynders J.H. García-Palacios G.C. Marano B. Briseghella G. De Roeck Wireless-based identification and model updating of a skewed highway bridge for structural health monitoring Appl. Sci. 10 2020 doi: 10.3390/app10072347.
[6] Sun L, Li Y, Zhang W. Experimental Study on Continuous Bridge-Deflection Estimation through Inclination and Strain. J Bridg Eng 2020;25:04020020. https://doi.org/10.1061/(asce)be.1943-5592.0001543.
[7] Farshadi M, Esfandiari A, Vahedi M. Structural model updating using incomplete transfer function and modal data. Struct Control Heal Monit 2017;24:1–13. https://doi.org/10.1002/stc.1932.
[8] Sun L, Li Y, Zhu W, Zhang W. Structural response reconstruction in physical coordinate from deficient measurements. Eng Struct 2020;212:110484. https://doi.org/10.1016/j.engstruct.2020.110484.
[9] Giagopoulos D, Arailopoulos A, Dertimanis V, Papadimitriou C, Chatzi E, Grompanopoulos K. Structural health monitoring and fatigue damage estimation using vibration measurements and finite element model updating. Struct Heal Monit 2019;18:1189–206. https://doi.org/10.1177/1475921718790188.
[10] Kaveh A, Davaran A. Spectral bisection of adaptive finite element meshes for parallel processing. Comput Struct 1999;70:315–23. https://doi.org/10.1016/S0045-7949(98)00170-9.
[11] Kaveh A, Rahimi Bondarabady HA. Spectral trisection of finite element models. Int J Numer Methods Heat Fluid Flow 2001;11:358–70. https://doi.org/10.1108/09615530110389199.
[12] Kaveh A, Rahimi Bondarabady HAA. multi-level finite element nodal ordering using algebraic graph theory. Finite Elem Anal Des 2002;38:245–61. https://doi.org/10.1016/S0168-874X(01)00062-2.
[13] A. Depeursinge D. Racoceanu J. Iavindrasana G. Cohen A. Platon P.-A. Poletti et al. Fusing Visual and Clinical Information for Lung Tissue Classification in HRCT Data Artif. Intell. Med. 40 2010 ARTMED1118 10.1016/j.
[14] Kaveh A, Roosta GR. Domain decomposition for finite element analysis. Commun Numer Methods Eng 1997;13:61–71. https://doi.org/10.1002/(SICI)1099-0887(199702)13:2<61::AID-CNM30>3.0.CO;2-6.
[15] Garg R.P.,Sharapov I. Techniques for Optimizing Applications: High Performance Computing. 2001; xliv + 616. books/apt.pdf;%5Cnhttp://www.sun.com/books/catalog/garg.html/index.html.
[16] Helfenstein R, Koko J. Parallel preconditioned conjugate gradient algorithm on GPU. J Comput Appl Math 2012;236:3584–90. https://doi.org/10.1016/j.cam.2011.04.025.
[17] S. Heydari S.A. Gharebaghi Modal analysis of two-dimensional beams using parallel finite-element method Sci. Iran. 24 2017 2762 2775 https://doi.org/10.24200/sci.2017.4529.
[18] Molina-Moya J, Martínez-Castro AE, Ortiz P. An iterative parallel solver in GPU applied to frequency domain linear water wave problems by the boundary element method. Front Built Environ 2018;4:1–11. https://doi.org/10.3389/fbuil.2018.00069.
[19] Herrera JFR, Salmerón JMG, Hendrix EMT, Asenjo R, Casado LG. On parallel Branch and Bound frameworks for Global Optimization. J Glob Optim 2017;69:547–60. https://doi.org/10.1007/s10898-017-0508-y.
[20] Marwala T, Boulkaibet I, Adhikari S. Probabilistic Finite Element Modle Updating Using Bayesian Statistics. 1st editio,. Wiley, United Kingdom; 2017.
[21] Aruna A, Ganguli R. Multi-fidelity response surface for uncertainty quantification in beams using coarse and fine finite element discretizations. Int J Comput Methods Eng Sci Mech 2021;22:103–22.
[22] Kaveh A, Fazli H. Graph coloration and group theory in dynamic analysis of symmetric finite element models. Finite Elem Anal Des 2007;43:901–11. https://doi.org/10.1016/j.finel.2007.06.002.
[23] Kaveh A, Fazli H. Graph coloration and group theory for factorization of symmetric dynamic systems. Acta Mech 2007;192:111–33. https://doi.org/10.1007/s00707-006-0414-8.
[24] Kaveh A, Rahami H, Shojaei I, Swift. Analysis of civil engineering structures using graph theory methods. Cham, Switzerland: Springer; 2020.
[25] Kaveh A, Rahimi Bondarabady HA. Bisection for parallel computing using Ritz and Fiedler vectors. Acta Mech 2004;167:131–44. https://doi.org/10.1007/s00707-003-0070-1.
[26] Chen H-P, Ni Y-Q, editors. Structural Health Monitoring of Large Civil Engineering Structures. Chichester, UK: John Wiley & Sons, Ltd; 2018.
[27] Marwala T. Finite-element model Updating Using Computational Intelligence Techniques. 1st editio,. London, London: Springer-Verlag; 2010.
[28] Dhandole S, Modak SV. A constrained optimization based method for acoustic finite element model updating of cavities using pressure response. Appl Math Model 2012;36:399–413. https://doi.org/10.1016/j.apm.2011.07.029.
[29] Mottershead JE, Link M, Friswell MI. The sensitivity method in finite element model updating: A tutorial. Mech Syst Signal Process 2011;25:2275–96. https://doi.org/10.1016/j.ymssp.2010.10.012.
[30] Marwala T, Boulkaibet I, Adhikari S. Probabilistic Finite Element Model Updating using Bayesian Statistics. Application to aeronautical and mechanical engineering. 1st editio,. Chichester: John Wiley & Sons Ltd; 2017.
[31] Shahbaznia M, Dehkordi MR, Mirzaee A. An improved time-domain damage detection method for railway bridges subjected to unknown moving loads. Period Polytech Civ Eng 2020;64:928–38. https://doi.org/10.3311/PPci.15813.
[32] Schommer S, Nguyen VH, Maas S, Zürbes A. Model updating for structural health monitoring using static and dynamic measurements. Procedia Eng 2017;199:2146–53. https://doi.org/10.1016/j.proeng.2017.09.156.
[33] Friswell MI, Mottershead JE. Finite element model updating in structural dynamics. 1st editio,. Netherlands, Waterloo: Springer; 1995.
[34] Mottershead JE, Friswell MI. Model Updating in structural dynamics. J Sound Vib 1993;167:347–75.
[35] Bianconi F, Salachoris GP, Clementi F, Lenci S. A genetic algorithm procedure for the automatic updating of fem based on ambient vibration tests. Sensors (Switzerland) 2020;20:1–17. https://doi.org/10.3390/s20113315.
[36] Ye S, Lai X, Bartoli I, Aktan AE. Technology for condition and performance evaluation of highway bridges. J Civ Struct Heal Monit 2020;10:573–94. https://doi.org/10.1007/s13349-020-00403-6.
[37] Feng Y, Kaya Y, Ventura C. Finite element model updating of portage Creek Bridge. Conf Proc Soc Exp Mech Ser 2016;2:247–53. https://doi.org/10.1007/978-3-319-29751-4_25.
[38] Živanović S, Pavic A, Reynolds P. Finite element modelling and updating of a lively footbridge: The complete process. J Sound Vib 2007;301:126–45. https://doi.org/10.1016/j.jsv.2006.09.024.
[39] Altunişik AC, Okur FY, Genç AF, Günaydin M, Adanur S. Automated Model Updating of Historical Masonry Structures Based on Ambient Vibration Measurements. J Perform Constr Facil 2018;32(1):04017126.
[40] Altunişik AC, Karahasan OŞ, Okur FY, Kalkan E, Ozgan K. Finite element model updating and dynamic analysis of a restored historical timber mosque based on ambient vibration tests. J Test Eval 2019;47(5):20180122.
[41] Lacanna G, Betti M, Ripepe M, Bartoli G. Dynamic Identification as a Tool to Constrain Numerical Models for Structural Analysis of Historical Buildings. Front Built Environ 2020;6:1–13. https://doi.org/10.3389/fbuil.2020.00040.
[42] Altunişik AC, Okur FY, Genç AF, Günaydin M, Adanur S. Automated Model Updating of Historical Masonry Structures Based on Ambient Vibration Measurements. J Perform Constr Facil 2018;32:04017126. https://doi.org/10.1061/(asce)cf.1943-5509.0001108.
[43] Baruch M. Optimization procedure to correct stiffness and flexibility matrices using vibration tests. AIAA J 1978;16:1208–10. https://doi.org/10.2514/3.61032.
[44] Berman A, Nagy EJ. Improvement of a large analytical model using test data. AIAA J 1983;21:1168–73. https://doi.org/10.2514/3.60140.
[45] Jull MØØ, Amador SDR, Skafte A, Hansen JB, Aenlle ML, Brincker R. One-Step FE Model Updating Using Local Correspondence and Mode Shape Orthogonality. Shock Vib 2019;2019:1–12.
[46] Ewins DJ. Modal Testing: Theory, Practice and Application. 2nd editio,. Hertfordshire: Research Studies Press Ltd; 2000.
[47] Sen S.,Bhattacharya B. Eigen structure assignment based finite element model updating, in: Int. Conf. Comput. Aided Eng., Chennai, 2013.
[48] Asma F. Finite element model updating using Lagrange interpolation. Mech Mech Eng 2019;23:228–32. https://doi.org/10.2478/mme-2019-0030.
[49] M. Girardi C. Padovani D. Pellegrini M. Porcelli L. Robol Finite element model updating for structural applications J. Comput. Appl. Math. 270 2020 https://doi.org/https://doi.org/10.1016/j.cam.2019.112675.
[50] Heo G, Jeon J. An Experimental Study of Structural Identification of Bridges Using the Kinetic Energy Optimization Technique and the Direct Matrix Updating Method. Shock Vib 2016;2016:1–13.
[51] Eskew EL, Jang S. Remaining stiffness estimation of buildings using incomplete measurements. Struct Control Heal Monit 2017;24:1–11. https://doi.org/10.1002/stc.1899.
[52] Kaveh A, Ghaderi I. Conditioning of structural stiffness matrices. Comput Struct 1997;63:719–25. https://doi.org/10.1016/S0045-7949(96)00073-9.
[53] Kaveh A, Pishghadam M, Jafarvand A. Topology optimization of repetitive near-regular shell structures using preconditioned conjugate gradients method. Mech Based Des Struct Mach 2022;50(4):1434–55.
[54] Kaveh A. Optimizing the conditioning of structural flexibility matrices. Comput Struct 1991;41:489–94. https://doi.org/10.1016/0045-7949(91)90142-9.
[55] A. Kaveh Optimal analysis of structures by concepts of symmetry and regularity 2013 Springer London, England 10.1007/978-3-7091-1565-7.
[56] Beck JL, Yuen KV. Model Selection Using Response measurements: Bayesian Probabilistic Approach. J Eng Mech 2004;130(2):192–203. https://doi.org/10.1061/(ASCE)0733-9399(2004)130.
[57] Titurus B, Friswell MI. Regularization in model updating. Int J Numer Methods Eng 2008;75:440–78. https://doi.org/10.1002/nme.2257.
[58] Yuan Z, Liang P, Silva T, Yu K, Mottershead JE. Parameter selection for model updating with global sensitivity analysis. Mech Syst Signal Process 2019;115:483–96. https://doi.org/10.1016/j.ymssp.2018.05.048.
[59] Jang J, Smyth AW. Model updating of a full-scale FE model with nonlinear constraint equations and sensitivity-based cluster analysis for updating parameters. Mech Syst Signal Process 2017;83:337–55. https://doi.org/10.1016/j.ymssp.2016.06.018.
[60] Subset MAJ. Selection in Regression. 2nd editio,. New Delhi: Springer; 1995.
[61] Bruneau P, Parisot O, Otjacques B. A heuristic for the automatic parametrization of the spectral clustering algorithm. Proc - Int Conf Pattern Recognit 2014;2:1313–8. https://doi.org/10.1109/ICPR.2014.235.

[62] Silva TAN, Maia NMM, Link M, Mottershead JE. Parameter selection and covariance updating. Mech Syst Signal Process 2016;70–71:269–83. https://doi.org/10.1016/j.ymssp.2015.08.034.

[63] P. Asadollahi Y. Huang J. Li Bayesian finite element model updating and assessment of cable-stayed bridges using wireless sensor data Sensors (Switzerland) 18 2018 doi: 10.3390/s18093057.

[64] Li Y, Xu L. Unweighted multiple group method with arithmetic mean. Proc 5th Int Conf Bio-Inspired Comput Theor Appl 2010;100:830–4. https://doi.org/10.1109/BICTA.2010.5645232.

[65] Chiachío J, Chiachío M, Saxena A, Sankararaman S, Rus G, Goebel K. Bayesian model selection and parameter estimation for fatigue damage progression models in composites. Int J Fatigue 2015;70:361–73. https://doi.org/10.1016/j.ijfatigue.2014.08.003.

[66] Mthembu L.,Marwala T.,Friswell M.I.,Adhikari S. Finite element model selection using Particle Swarm Optimization, in: Int. Modal Anal. Conf., 2010.

[67] Arora V, Van Der Hoogt PJM, Aarts RGKM, De Boer A. Identification of stiffness and damping characteristics of axial air-foil bearings. Int J Mech Mater Des 2011; 7:231–43. https://doi.org/10.1007/s10999-011-9161-7.

[68] Brownjohn JMW. Structural health monitoring of civil infrastructure. Philos Trans R Soc A Math Phys Eng Sci 2007;365:589–622. https://doi.org/10.1098/rsta.2006.1925.

[69] Marwala T, Boulkaibet I, Adhikari S. Probabilistic Finite Element Model Updating using Bayesian Statistics. Application to aeronautical and mechanical engineering. Chichester, UK: Ltd Registered; 2017.

[70] Jang J, Smyth A. Bayesian model updating of a full-scale finite element model with sensitivity-based clustering. Struct Control Heal Monit 2017;24:1–15. https://doi.org/10.1002/stc.2004.

[71] Yuen KV. Recent developments of Bayesian model class selection and applications in civil engineering. Struct Saf 2010;32:338–46. https://doi.org/10.1016/j.strusafe.2010.03.011.

[72] Yin T, Zhu HP, Fu SJ. Model selection for dynamic reduction-based structural health monitoring following the Bayesian evidence approach. Mech Syst Signal Process 2019;127:306–27. https://doi.org/10.1016/j.ymssp.2019.03.009.

[73] Durmazgezer E, Yucel U, Ozcelik O. Damage identification of a reinforced concrete frame at increasing damage levels by sensitivity-based finite element model updating. Bull Earthq Eng 2019;17:6041–60. https://doi.org/10.1007/s10518-019-00690-5.

[74] Nozari A, Behmanesh I, Yousefianmoghadam S, Moaveni B, Stavridis A. Effects of variability in ambient vibration data on model updating and damage identification of a 10-story building. Eng Struct 2017;151:540–53. https://doi.org/10.1016/j.engstruct.2017.08.044.

[75] Y. Wu R. Zhu Z. Cao Y. Liu D. Jiang Model updating using frequency response functions based on sherman-morrison formula Appl. Sci. 10 2020 doi: 10.3390/app10144985.

[76] Davis NT, Sanayei M. Foundation identification using dynamic strain and acceleration measurements. Eng Struct 2020;208:109811.

[77] Jaishi B, Ren W-X. Structural Finite Element Model Updating Using Ambient Vibration Test Results. J Struct Eng 2005;131:617–28. https://doi.org/10.1061/(asce)0733-9445(2005)131:4(617).

[78] M. Razavi A. Hadidi Assessment of sensitivity-based FE model updating technique for damage detection in large space structures Struct. Monit. Maint. 7 2020 261 281 https://doi.org/10.12989/smm.2020.7.3.261.

[79] Jaishi B, Ren WX. Finite element model updating based on eigenvalue and strain energy residuals using multiobjective optimisation technique. Mech Syst Signal Process 2007;21:2295–317. https://doi.org/10.1016/j.ymssp.2006.09.008.

[80] Yang X, Ouyang H, Guo X, Cao S. Modal Strain Energy-Based Model Updating Method for Damage Identification on Beam-Like Structures. J Struct Eng 2020; 146:04020246. https://doi.org/10.1061/(asce)st.1943-541x.0002812.

[81] Liao J, Tang G, Meng L, Liu H, Zhang Y. Finite element model updating based on field quasi-static generalized influence line and its bridge engineering application. Procedia Eng 2012;31:348–53. https://doi.org/10.1016/j.proeng.2012.01.1035.

[82] Tchemodanova SP, Sanayei M, Moaveni B, Tatsis K, Chatzi E. Strain predictions at unmeasured locations of a substructure using sparse response-only vibration measurements. J Civ Struct Heal Monit 2021;11(4):1113–36.

[83] Kim S, Young K, Lee J. Bridge Finite Model Updating Approach By Static Load Input / Deflection Output Measurements: Field Experiment. Korea: Jeju Island; 2016.

[84] Sanayei M, Imbaro GR, McClain JAS, Brown LC. Structural Model Updating Using Experimental Static Measurements. J Struct Eng 1997;123:792–8. https://doi.org/10.1061/(asce)0733-9445(1997)123:6(792).

[85] Zhou Y, Zhang J, Yi W, Jiang Y, Pan Q. Structural Identification of a Concrete-Filled Steel Tubular Arch Bridge via Ambient Vibration Test Data. J Bridg Eng 2017;22:04017049. https://doi.org/10.1061/(asce)be.1943-5592.0001086.

[86] Nazarian E, Ansari F, Azari H. Recursive optimization method for monitoring of tension loss in cables of cable-stayed bridges. J Intell Mater Syst Struct 2016;27: 2091–101. https://doi.org/10.1177/1045389X15620043.

[87] L. Sun Y. Xu Modal parameter identification and finite element model updating of a long-span aqueduct structure based on ambient excitation J. Vibroengineering 22 2020 896 908 https://doi.org/10.21595/jve.2020.21271.

[88] Sanayei M, Bell ES, Javdekar CN, Edelmann JL, Slavsky E. Damage Localization and Finite-Element Model Updating Using Multiresponse NDT Data. J Bridg Eng 2006;11:688–98. https://doi.org/10.1061/(asce)1084-0702(2006)11:6(688).

[89] Sipple JD, Sanayei M. Finite element model updating of the UCF grid benchmark using measured frequency response functions. Mech Syst Signal Process 2014;46: 179–90. https://doi.org/10.1016/j.ymssp.2014.01.008.

[90] Bandara RP, Chan TH, Thambiratnam DP. Structural damage detection method using frequency response functions. Struct Heal Monit 2014;13:418–29. https://doi.org/10.1177/1475921714522847.

[91] Pu Q, Hong Y, Chen L, Yang S, Xu X. Model updating–based damage detection of a concrete beam utilizing experimental damped frequency response functions. Adv Struct Eng 2019;22:935–47. https://doi.org/10.1177/1369433218789556.

[92] Wang J, Wang C, Updating SM, of Frequency Response Function Based on Kriging Model. Proc. -,. 3rd Int. Conf Inf Sci Control Eng ICISCE 2016;2016(2016):640–4. https://doi.org/10.1109/ICISCE.2016.142.

[93] Pradhan S, Modak SV. Damping Matrix Identification by Finite Element Model Updating Using Frequency Response Data. Int J Struct Stab Dyn 2017;17:1–27. https://doi.org/10.1142/S0219455417500043.

[94] Oh BK, Kim D, Park HS. Modal Response-Based Visual System Identification and Model Updating Methods for Building Structures. Comput Civ Infrastruct Eng 2017;32:34–56. https://doi.org/10.1111/mice.12229.

[95] Wu WH, Prendergast LJ, Gavin K. An iterative method to infer distributed mass and stiffness profiles for use in reference dynamic beam-Winkler models of foundation piles from frequency response functions. J Sound Vib 2018;431:1–19. https://doi.org/10.1016/j.jsv.2018.05.049.

[96] Zhou S, Song W. Environmental-effects-embedded model updating method considering environmental impacts. Struct Control Heal Monit 2018;25:1–22. https://doi.org/10.1002/stc.2116.

[97] Cui J, Kim D, Koo KY, Chaudhary S. Structural model updating of steel box girder bridge using modal flexibility based deflections. Balt J Road Bridg Eng 2012;7: 253–60. https://doi.org/10.3846/bjrbe.2012.34.

[98] Dinh-Cong D, Nguyen-Thoi T, Nguyen DT. A FE model updating technique based on SAP2000-OAPI and enhanced SOS algorithm for damage assessment of full-scale structures. Appl Soft Comput J 2020;89:106100. https://doi.org/10.1016/j.asoc.2020.106100.

[99] Rezaiee-Pajand M, Entezami A, Sarmadi H. A sensitivity-based finite element model updating based on unconstrained optimization problem and regularized solution methods. Struct Control Heal Monit 2020;27:1–29. https://doi.org/10.1002/stc.2481.

[100] Wang FL, Chan THT, Thambiratnam DP, Tan ACC. Damage diagnosis for complex steel truss bridges using multi-layer genetic algorithm. J Civ Struct Heal Monit 2013;3:117–27. https://doi.org/10.1007/s13349-013-0041-8.

[101] Srinivas V, Ramanjaneyulu K, Jeyasehar CA. Multi-stage approach for structural damage identification using modal strain energy and evolutionary optimization techniques. Struct Heal Monit 2011;10:219–30. https://doi.org/10.1177/1475921710373291.

[102] Özer E, Soyöz S. Vibration-based damage detection and seismic performance assessment of bridges. Earthq Spectra 2015;31:137–57. https://doi.org/10.1193/080612EQS255M.

[103] Li J, Hao H, Chen Z. Damage Identification and Optimal Sensor Placement for Structures under Unknown Traffic-Induced Vibrations. J Aerosp Eng 2017;30: 1–10. https://doi.org/10.1061/(asce)as.1943-5525.0000550.

[104] Feng D, Feng MQ. Model Updating of Railway Bridge Using In Situ Dynamic Displacement Measurement under Trainloads. J Bridg Eng 2015;20:04015019. https://doi.org/10.1061/(asce)be.1943-5592.0000765.

[105] Wang H, Li AQ, Li J. Progressive finite element model calibration of a long-span suspension bridge based on ambient vibration and static measurements. Eng Struct 2010;32:2546–56. https://doi.org/10.1016/j.engstruct.2010.04.028.

[106] Erdogan YS, Gul M, Catbas FN, Bakir PG. Investigation of Uncertainty Changes in Model Outputs for Finite-Element Model Updating Using Structural Health Monitoring Data. J Struct Eng 2014;140:04014078. https://doi.org/10.1061/(asce)st.1943-541x.0001002.

[107] Sanayei M, Khaloo A, Gul M, Necati CF. Automated finite element model updating of a scale bridge model using measured static and modal test data. Eng Struct 2015;102:66–79. https://doi.org/10.1016/j.engstruct.2015.07.029.

[108] S. Kim N. Kim Y.-S. Park S.-S. Jin A Sequential Framework for Improving Identifiability of FE Model Updating Using Static and Dynamic Data Sensors (Switzerland) 19 2019 https://doi.org/doi:10.3390/s19235099.

[109] Schlune H, Plos M, Gylltoft K. Improved bridge evaluation through finite element model updating using static and dynamic measurements. Eng Struct 2009;31: 1477–85. https://doi.org/10.1016/j.engstruct.2009.02.011.

[110] Jiménez-Alonso JF, Naranjo-Perez J, Pavic A, Sáez A. Maximum Likelihood Finite-Element Model Updating of Civil Engineering Structures Using Nature-Inspired Computational Algorithms. Struct Eng Int 2021;31(3):326–38.

[111] Goller B, Beck JL, Schuëller GI. Evidence-Based Identification of Weighting Factors in Bayesian Model Updating Using Modal Data. J Eng Mech 2012;138: 430–40. https://doi.org/10.1061/(asce)em.1943-7889.0000351.

[112] F. Pacheco-Torgal R. Melchers X. Shi N. De Belie K. Van Tittelboom A. Saez Eco-efficient Repair and Rehabilitation of Concrete Infrastructure 2017 Jonathan Simpson, Kidlington, UK.

[113] Cha YJ, Buyukozturk O. Structural damage detection using modal strain energy and hybrid multiobjective optimization. Comput Civ Infrastruct Eng 2015;30: 347–58. https://doi.org/10.1111/mice.12122.

[114] Osyczka A. An approach to multicriterion optimization problems for engineering design. Comput Methods Appl Mech Eng 1978;15:309–33. https://doi.org/10.1016/0045-7825(78)90046-4.

[115] Cuate O, Schütze O. Pareto explorer for finding the knee for many objective optimization problems. Mathematics 2020;8:6–15. https://doi.org/10.3390/MATH8101651.

[116] M. Nagy Y. Mansour S. Abdelmohsen Multi-Objective Optimization Methods as a Decision Making Strategy Int. J. Eng. Res. V9 2020 https://doi.org/10.17577/ijertv9is030480.

[117] Branke J, Deb K, Dierolf H, Osswald M. Finding knees in multi-objective optimization. Lect Notes Comput Sci 2004:722–31. https://doi.org/10.1007/978-3-540-30217-9_73.
[118] Jin SS, Cho S, Jung HJ, Lee JJ, Yun CB. A new multi-objective approach to finite element model updating. J Sound Vib 2014;333:2323–38. https://doi.org/10.1016/j.jsv.2014.01.015.
[119] Naranjo-Pérez J, Jiménez-Alonso JF, Pavic A, Sáez A. Finite-element-model updating of civil engineering structures using a hybrid UKF-HS algorithm. Struct Infrastruct Eng 2021;17:620–37. https://doi.org/10.1080/15732479.2020.1760317.
[120] Mthembu L, Marwala T, Friswell MI, Adhikari S. Model selection in finite element model updating using the Bayesian evidence statistic. Mech Syst Signal Process 2011;25:2399–412. https://doi.org/10.1016/j.ymssp.2011.04.001.
[121] Ghaderinezhad F.,Ley C. On the Impact of Choice of the Prior in Bayesian Statistics, in: N. Tang (Ed.), Bayesian Inference Complicat. Data, 2020. https://doi.org/DOI: 10.5772/intechopen.83214.
[122] Das A, Debnath N. A Bayesian finite element model updating with combined normal and lognormal probability distributions using modal measurements. Appl Math Model 2018;61:457–83. https://doi.org/10.1016/j.apm.2018.05.004.
[123] Mthembu L, Marwala T, Friswell MI, Adhikari S. Bayesian evidence for finite element model updating. Ser: Conf. Proc. Soc. Exp. Mech; 2009.
[124] Jia X, Papadimitriou C. Data features-based likelihood-informed Bayesian finite element model updating. Proc 3rd Int Conf Uncertain Quantif Comput Sci Eng UNCECOMP 2019:103–13. https://doi.org/10.7712/120219.6328.18902.
[125] Khodaparast HH, Mottershead JE, Friswell MI. Perturbation methods for the estimation of parameter variability in stochastic model updating. Mech Syst Signal Process 2008;22:1751–73. https://doi.org/10.1016/j.ymssp.2008.03.001.
[126] Das A, Debnath N. A Bayesian model updating with incomplete complex modal data. Mech Syst Signal Process 2020;136:106524. https://doi.org/10.1016/j.ymssp.2019.106524.
[127] T. Marwala Finite-element-model updating using computional intelligence techniques: Applications to structural dynamics 1st editio, 2010 Springer-Verlag London, London, England 10.1007/978-1-84996-323-7.
[128] Huang Y, Beck JL, Li H. Hierarchical sparse Bayesian learning for structural damage detection: Theory, computation and application. Struct Saf 2017;64:37–53. https://doi.org/10.1016/j.strusafe.2016.09.001.
[129] Prakash G, Narasimhan S. Bayesian Two-Phase Gamma Process Model for Damage Detection and Prognosis. J Eng Mech 2018;144:04017158. https://doi.org/10.1061/(asce)em.1943-7889.0001386.
[130] Brooks S, Gelman A, Carlin JB, Stern HS, Rubin DB. Bayesian Data Analysis. 3rd editio,. Boca Raton, USA: Taylor & Francis; 1996.
[131] Grazian C, Robert CP. Jeffreys priors for mixture estimation: Properties and alternatives. Comput Stat Data Anal 2018;121:149–63. https://doi.org/10.1016/j.csda.2017.12.005.
[132] Yan W, Hu S, Yang Y, Gao F, Chen T. Bayesian migration of Gaussian process regression for rapid process modeling and optimization. Chem Eng J 2011;166:1095–103. https://doi.org/10.1016/j.cej.2010.11.097.
[133] Berger JO, Bernardo JM, Sun D. The formal definition of reference priors. Ann Stat 2009;37:905–38. https://doi.org/10.1214/07-AOS587.
[134] Pooley CM, Marion G. Bayesian model evidence as a practical alternative to deviance information criterion. R Soc Open Sci 2018;5(3):171519.
[135] Eltouny K.,Liang X. A nonparametric unsupervised learning approach for structural damage detection. ArXiv 2020.
[136] Z. Feng Y. Lin W. Wang X. Hua Z. Chen Probabilistic Updating of Structural Models for Damage Assessment Using Approximate Bayesian Computation Sensors (Switzerland) 20 2020 https://doi.org/doi:10.3390/s20113197.
[137] Song M, Behmanesh I, Moaveni B, Papadimitriou C. Modeling error estimation and response prediction of a 10-story building model through a hierarchical bayesian model updating framework. Front Built Environ 2019;5. https://doi.org/10.3389/fbuil.2019.00007.
[138] Lye A, Cicirello A, Patelli E. Sampling methods for solving Bayesian model updating problems: A tutorial. Mech Syst Signal Process 2021;159:1–104. https://doi.org/10.1016/j.ymssp.2021.107760.
[139] Govers Y, Link M. Stochastic model updating-Covariance matrix adjustment from uncertain experimental modal data. Mech Syst Signal Process 2010;24:696–706. https://doi.org/10.1016/j.ymssp.2009.10.006.
[140] Soize C. Random matrix theory for modeling uncertainties in computational mechanics. Com- Puter Methods Appl Mech Eng 2012;194:1333–66. https://doi.org/10.1016/j.cma.2004.06.038.
[141] Alvin KF. Finite element model update via bayesian estimation and minimization of dynamic residuals. AIAA J 1997;35:879–86. https://doi.org/10.2514/2.7462.
[142] R. Moore R.B. Kearfott M.J. Cloud Interval, Analysis. Society for industrial and Applied Mathematics 2009 Philadelphia 10.1007/1-84628-108-3_18.
[143] Zadeh LA. Fuzzy Sets. Inf Control 1965;8(3):338–53.
[144] Liu Y, Duan Z. Fuzzy finite element model updating of bridges by considering the uncertainty of the measured modal parameters. Sci China Technol Sci 2012;55:3109–17. https://doi.org/10.1007/s11431-012-5009-0.
[145] Bulkaibeto I, Marwala T, Friswell MI, Khodaparast HH, Adhikari S. Fuzzy Finite Element Model Updating Using Metaheuristic Optimization Algorithms 2017:91–101. https://doi.org/10.1007/978-3-319-53841-9_8.
[146] T. Banakh Classical Set Theory: Theory of Sets and Classes. 2020 1 162 http://arxiv.org/abs/2006.01613.
[147] Xiao Z, Xia S, Gong K, Li D. The trapezoidal fuzzy soft set and its application in MCDM. Appl Math Model 2012;36:5844–55. https://doi.org/10.1016/j.apm.2012.01.036.
[148] Mohan BM. Discussion on Mathematical Modeling of Fuzzy Two-Term (PI / PD). Controllers 2010;8:38–40.
[149] Viattchenin DA, Tati R, Damaratski A. Designing Gaussian membership functions for fuzzy classifier generated by heuristic possibilistic clustering. J Inf Organ Sci 2013;37:127–39.
[150] Słowiński R. Fuzzy Sets in Decision Analysis, Operations Research and Statistics. 1st editio,. LLC, New York, USA: Springer science+ business media; 1998.
[151] Qiu Z, Hu J, Yang J, Lu Q. Exact bounds for the sensitivity analysis of structures with uncertain-but-bounded parameters. Appl Math Model 2008;32:1143–57. https://doi.org/10.1016/j.apm.2007.03.004.
[152] Jiang D.,Zhang P.,Fei Q.,Wu S. 1330. Comparative study of model updating methods using frequency response function data. J. Vibroengineering 2014; 16; 2305–2318.
[153] Zhu Q, Xu YL, Xiao X. Multiscale Modeling and Model Updating of a Cable-Stayed Bridge. I: Modeling and Influence Line Analysis. J Bridg Eng 2015;20:04014112. https://doi.org/10.1061/(asce)be.1943-5592.0000722.
[154] Ren WX, Chen HB. Finite element model updating in structural dynamics by using the response surface method. Eng Struct 2010;32:2455–65. https://doi.org/10.1016/j.engstruct.2010.04.019.
[155] Park W, Kim HK, Jongchil P. Finite element model updating for a cable-stayed bridge using manual tuning and sensitivity-based optimization. Struct Eng Int J Int Assoc Bridg Struct Eng 2012;22:14–9. https://doi.org/10.2749/101686612X13216060212870.
[156] J. Li H. Hao J.V. Lo Structural damage identification with power spectral density transmissibility: Numerical and experimental studies Smart Struct. Syst. 15 2015 15 40 https://doi.org/10.12989/sss.2015.15.1.015.
[157] Venanzi I, Kita A, Cavalagli N, Ierimonti L, Ubertini F. Earthquake-induced damage localization in an historic masonry tower through long-term dynamic monitoring and FE model calibration. Bull Earthq Eng 2020;18:2247–74. https://doi.org/10.1007/s10518-019-00780-4.
[158] Pianosi F, Beven K, Freer J, Hall JW, Rougier J, Stephenson DB, et al. Sensitivity analysis of environmental models: A systematic review with practical workflow. Environ Model Softw 2016;79:214–32. https://doi.org/10.1016/j.envsoft.2016.02.008.
[159] Jaishi B, Ren WX. Damage detection by finite element model updating using modal flexibility residual. J Sound Vib 2006;290:369–87. https://doi.org/10.1016/j.jsv.2005.04.006.
[160] Entezami A, Shariatmadar H, Sarmadi H. Structural damage detection by a new iterative regularization method and an improved sensitivity function. J Sound Vib 2017;399:285–307. https://doi.org/10.1016/j.jsv.2017.02.038.
[161] Blachowski B. Modal sensitivity based sensor placement for damage identification under sparsity constraint. Period Polytech Civ Eng 2019;63:432–45. https://doi.org/10.3311/PPci.13888.
[164] Waseda University S. Darwin ' s Theory of Evolution by Natural Selection The Facts about Evolution. no date 1–31.
[165] M. Clerc Particle Swarm Optimization 2006 ISTE London, UK.
[166] Bassoli E, Vincenzi L, D'Altri AM, de Miranda S, Forghieri M, Castellazzi G. Ambient vibration-based finite element model updating of an earthquake-damaged masonry tower. Struct Control Heal Monit 2018;25:1–15. https://doi.org/10.1002/stc.2150.
[167] Jahangiri M, Najafgholipour MA, Dehghan SM, Hadianfard MA. The efficiency of a novel identification method for structural damage assessment using the first vibration mode data. J Sound Vib 2019;458:1–16. https://doi.org/10.1016/j.jsv.2019.06.011.
[168] H. Tran-Ngoc S. Khatir G. De Roeck T. Bui-Tien L. Nguyen-Ngoc W.M. Abdel Model updating for nam O bridge using particle swarm optimization algorithm and genetic algorithm Sensors (Switzerland) 18 2018 doi: 10.3390/s18124131.
[169] Qin S, Zhou YL, Cao H, Wahab MA. Model Updating in Complex Bridge Structures using Kriging Model Ensemble with Genetic Algorithm. KSCE J Civ Eng 2018;22:3567–78. https://doi.org/10.1007/s12205-017-1107-7.
[170] Raich AM, Liszkai TR. Improving the performance of structural damage detection methods using advanced genetic algorithms. J Struct Eng 2007;133(3):449–61.
[171] Alkayem NF, Cao M, Ragulskis M. Damage localization in irregular shape structures using intelligent FE model updating approach with a new hybrid objective function and social swarm algorithm. Appl Soft Comput J 2019;83:105604.
[172] Shabbir F, Omenzetter P. Model updating using genetic algorithms with sequential niche technique. Eng Struct 2016;120:166–82. https://doi.org/10.1016/j.engstruct.2016.04.028.
[173] Nasr DE, Saad GA. Optimal Sensor Placement Using a Combined Genetic Algorithm-Ensemble Kalman Filter Framework. ASCE-ASME J. Risk Uncertain. Eng. Syst. Part A. Civ Eng 2017;3:04016010. https://doi.org/10.1061/ajrua6.0000886.
[174] Jiménez-Alonso JF, Sáez A. Model updating for the selection of an ancient bridge retrofitting method in Almeria. Spain Struct Eng Int 2016;26:17–26. https://doi.org/10.2749/101686615X14355644771333.
[175] Costa C, Ribeiro D, Jorge P, Silva R, Arêde A, Calçada R. Calibration of the numerical model of a stone masonry railway bridge based on experimentally identified modal parameters. Eng Struct 2016;123:354–71. https://doi.org/10.1016/j.engstruct.2016.05.044.
[176] Sabamehr A, Lim C, Bagchi A. System identification and model updating of highway bridges using ambient vibration tests. J Civ Struct Heal Monit 2018;8:755–71. https://doi.org/10.1007/s13349-018-0304-5.
[177] Pachón P, Castro R, García-Macías E, Compan V, Puertas EE. Torroja's bridge: Tailored experimental setup for SHM of a historical bridge with a reduced number

of sensors. Eng Struct 2018;162:11–21. https://doi.org/10.1016/j.engstruct.2018.02.035.
[178] Hernández-Díaz A.M.,Pérez-Aracil J.,Jiménez-Alonso J.F.,Sáez A. Self-control of a lively footbridge under pedestrian flow. no date 606–614.
[179] Gentilini C, Marzani A, Mazzotti M. Nondestructive characterization of tie-rods by means of dynamic testing, added masses and genetic algorithms. J Sound Vib 2013;332:76–101. https://doi.org/10.1016/j.jsv.2012.08.009.
[180] X. Yang X. Guo H. Ouyang D. Li A kriging model based finite element model updating method for damage detection Appl. Sci. 7 2017 doi: 10.3390/app7101039.
[181] H. Sun W. Chen S. Cai B. Zhang Mechanical State Assessment of In-Service Cable-Stayed Bridge Using a Two-Phase Model Updating Technology and Periodic Field Measurements J. Bridg. Eng. 25 2020 04020015 10.1061/(asce)be.1943-5592.0001550.
[182] Oh BK, Hwang JW, Choi SW, Kim Y, Cho T, Park HS. Dynamic displacements-based model updating with motion capture system. Struct Control Heal Monit 2017;24:1–16. https://doi.org/10.1002/stc.1904.
[183] Cui Y, Lu W, Teng J. Updating of structural multi-scale monitoring model based on multi-objective optimisation. Adv Struct Eng 2019;22:1073–88. https://doi.org/10.1177/1369433218805235.
[184] Wang FY, Xu YL, Zhan S. Multi-scale model updating of a transmission tower structure using Kriging meta-method. Struct Control Heal Monit 2017;24:1–16. https://doi.org/10.1002/stc.1952.
[185] Luong HTM, Zabel V, Lorenz W, Rohrmann RG. Vibration-based Model Updating and Identification of Multiple Axial Forces in Truss Structures. Procedia Eng 2017;188:385–92. https://doi.org/10.1016/j.proeng.2017.04.499.
[186] Mosquera V, Smyth AW, Betti R. Rapid evaluation and damage assessment of instrumented highway bridges: DAMAGE ASSESSMENT OF INSTRUMENTED HIGHWAY BRIDGES. Earthquake Engng Struct Dyn 2012;41(4):755–74.
[187] Seon PH, Kim JH, Oh BK. Model updating method for damage detection of building structures under ambient excitation using modal participation ratio. Meas J Int Meas Confed 2019;133:251–61. https://doi.org/10.1016/j.measurement.2018.10.023.
[188] P. Jeenkour J. Pattavanitch K. Boonlong Vibration-based damage detection in beams by genetic algorithm encoding locations and damage factors as decision variables Vibroengineering Procedia 16 2017 35 40 https://doi.org/10.21595/vp.2017.19345.
[189] Yu S, Ou J. Structural Health Monitoring and Model Updating of Aizhai Suspension Bridge. J Aerosp Eng 2017;30:1–15. https://doi.org/10.1061/(asce)as.1943-5525.0000653.
[190] R. Soman P. Mainowski A real-valued genetic algorithm for optimization of sensor placement for guided wave-based structural health monitoring J. Sensors 2019 1 10 https://doi.org/https://www.hindawi.com/journals/js/2019/9614630/.
[191] Hou R, Xia Y, Xia Q, Zhou X. Genetic algorithm based optimal sensor placement for L1-regularized damage detection. Struct Control Heal Monit 2019;26:1–14. https://doi.org/10.1002/stc.2274.
[192] Boonlong K. Vibration-based damage detection in beams by cooperative coevolutionary genetic algorithm. Adv Mech Eng 2014;6:624949.
[193] Pachón P, Infantes M, Cámara M, Compán V, García-Macías E, Friswell MI, et al. Evaluation of optimal sensor placement algorithms for the Structural Health Monitoring of architectural heritage. Application to the Monastery of San Jerónimo de Buenavista (Seville, Spain). Eng Struct 2020;202:109843.
[194] Okwu MO, Tartibu LK. Particle Swarm Optimisation. Stud. Comput Intell 2021; 927:5–13. https://doi.org/10.1007/978-3-030-61111-8_2.
[195] Gökdağ H, Yildiz AR. Structural damage detection using modal parameters and particle swarm optimization. Mater Test 2012;54:416–20. https://doi.org/10.3139/120.110346.
[196] Marwala T. Finite-element-model updating using computional intelligence techniques: Applications to structural dynamics. Finite-Element-Model Updat Using Comput Intell Tech Appl to Struct Dyn 2010:1–250. https://doi.org/10.1007/978-1-84996-323-7.
[197] Mohan SC, Maiti DK, Maity D. Structural damage assessment using FRF employing particle swarm optimization. Appl Math Comput 2013;219: 10387–400. https://doi.org/10.1016/j.amc.2013.04.016.
[198] Seyedpoor SM. A two stage method for structural damage detection using a modal strain energy based index and particle swarm optimization. Int J Non Linear Mech 2012;47:1–8. https://doi.org/10.1016/j.ijnonlinmec.2011.07.011.
[199] Nanda B, Maity D, Maiti DK. Crack assessment in frame structures using modal data and unified particle swarm optimization technique. Adv Struct Eng 2014;17: 747–66. https://doi.org/10.1260/1369-4332.17.5.747.
[200] Zhang X, Gao RX, Yan R, Chen X, Sun C, Yang Z. Multivariable wavelet finite element-based vibration model for quantitative crack identification by using particle swarm optimization. J Sound Vib 2016;375:200–16. https://doi.org/10.1016/j.jsv.2016.04.018.
[201] Gerist S, Maheri MR. Multi-stage approach for structural damage detection problem using basis pursuit and particle swarm optimization. J Sound Vib 2016; 384:210–26. https://doi.org/10.1016/j.jsv.2016.08.024.
[202] Nouri Shirazi MR, Mollamahmoudi H, Seyedpoor SM. Structural Damage Identification Using an Adaptive Multi-stage Optimization Method Based on a Modified Particle Swarm Algorithm. J Optim Theory Appl 2014;160:1009–19. https://doi.org/10.1007/s10957-013-0316-6.
[203] Perera R, Fang SE, Ruiz A. Application of particle swarm optimization and genetic algorithms to multiobjective damage identification inverse problems with modelling errors. Meccanica 2010;45:723–34. https://doi.org/10.1007/s11012-009-9264-5.
[204] Cancelli A, Laflamme S, Alipour A, Sritharan S, Ubertini F. Vibration-based damage localization and quantification in a pretensioned concrete girder using stochastic subspace identification and particle swarm model updating. Struct Heal Monit 2020;19:587–605. https://doi.org/10.1177/1475921718820015.
[205] Chatterjee S, Sarkar S, Hore S, Dey N, Ashour AS, Balas VE. Particle swarm optimization trained neural network for structural failure prediction of multistoried RC buildings. Neural Comput Appl 2017;28:2005–16. https://doi.org/10.1007/s00521-016-2190-2.
[206] Gao XZ, Govindasamy V, Xu H, Wang X, Zenger K. Harmony search method: Theory and applications. Comput Intell Neurosci 2015;2015:1–10.
[207] Kirkpatrick S, Gelatt CD, Vecchi MP. Optimization by simulated annealing. Science (80-) 1983;220(4598):671–80.
[208] Ghasemi S, Amiri GG, Dehcheshmeh MM. Structural damage assessment via model updating using augmented grey wolf optimization algorithm. Int J Eng Trans A Basics 2020;33:1173–82. https://doi.org/10.5829/ije.2020.33.07a.02.
[209] Kaveh A, Mahdavi VR. Damage identification of truss structures using CBO and ECBO algorithms. Asian J Civ Eng 2016;17:75–89.
[210] Rashedi E, Nezamabadi-pour H, Saryazdi S, Gsa. A Gravitational Search Algorithm. Inf Sci (Ny) 2009;179:2232–48. https://doi.org/10.1016/j.ins.2009.03.004.
[211] Vasuki A. Nature-inspired optimization algorithms. 1st editio,. New York: Taylor & Francis; 2020.
[212] Metropolis N, Rosenbluth AW, Rosenbluth MN, Teller AH, Teller E. Equation of state calculations by fast computing machines. J Chem Phys 1953;21:1087–92. https://doi.org/10.1063/1.1699114.
[213] Levin RI, Lieven NAJ. Dynamic finite element model updating using simulated annealing and genetic algorithms. Mech Syst Signal Process 1998;12:91–120. https://doi.org/10.1006/mssp.1996.0136.
[214] Marwala T. Finite element model updating using response surface method. Collect Tech Pap - AIAA/ASME/ASCE/AHS/ASC Struct Struct Dyn Mater Conf 2004;7: 5165–73. https://doi.org/10.2514/6.2004-2005.
[215] Kourehli SS. Damage diagnosis of structures using modal data and static response. Period Polytech Civ Eng 2017;61:135–45. https://doi.org/10.3311/PPci.7646.
[216] Zimmerman AT, Lynch JP. A Parallel Simulated Annealing Architecture for Model Updating in Wireless Sensor Networks. IEEE Sens J 2009;9:1503–10. https://doi.org/10.1109/JSEN.2009.2019323.
[217] Lam HF, Yang JH, Au SK. Markov chain Monte Carlo-based Bayesian method for structural model updating and damage detection. Struct Control Heal Monit 2018; 25:1–22. https://doi.org/10.1002/stc.2140.
[218] Huang Y, Shao C, Wu B, Beck JL, Li H. State-of-the-art review on Bayesian inference in structural system identification and damage assessment. Adv Struct Eng 2019;22:1329–51. https://doi.org/10.1177/1369433218811540.
[219] Green PL. Bayesian system identification of a nonlinear dynamica system using a novel variant of Simulated Annealing. Mech Syst Signal Process 2015;52–53: 133–46. https://doi.org/10.1016/j.ymssp.2014.07.010.
[220] Chiu PL, Lin FYS. A simulated annealing algorithm to support the sensor placement fot target location. Can Conf Electr Comput Eng 2004;2:0867–70. https://doi.org/10.1109/ccece.2004.1345252.
[221] Manjarres D, Landa-Torres I, Gil-Lopez S, Del Ser J, Bilbao MN, Salcedo-Sanz S, et al. A survey on applications of the harmony search algorithm. Eng Appl Artif Intell 2013;26:1818–31. https://doi.org/10.1016/j.engappai.2013.05.008.
[222] Zong Woo Geem, Joong Hoon Kim, Loganathan GV. A new heuristic optimization algorithm: Harmony search. A New Heuristic Optim. Algorithm Harmon. Search 2001;76(2):60–8.
[223] Long Q, Wu X, Wu C. Non-Dominated Sorting Methods for Multi-Objective Optimization: Review and Numerical Comparison. J Ind Manag Optim 2021;17: 1001–23. https://doi.org/10.3934/jimo.2020009.
[224] Liu J, Chen X. An improved NSGA-II algorithm based on crowding distance elimination strategy. Int J Comput Intell Syst 2019;12:513–8. https://doi.org/10.2991/ijcis.d.190328.001.
[225] Lee KS, Geem ZW. A new meta-heuristic algorithm for continuous engineering optimization: Harmony search theory and practice. Comput Methods Appl Mech Eng 2005;194:3902–33. https://doi.org/10.1016/j.cma.2004.09.007.
[226] Naranjo-Pérez J, Infantes M, Fernando Jiménez-Alonso J, Sáez A. A collaborative machine learning-optimization algorithm to improve the finite element model updating of civil engineering structures. Eng Struct 2020;225:111327.
[227] Kaveh A, Javadi SM, Maniat M. Damage assessment via modal data with a mixed particle swarm strategy, ray optimizer, and harmony search. Asian J Civ Eng 2014;15:95–106.
[228] Miguel LFF, Miguel LFF, Kaminski J, Riera JD. Damage detection under ambient vibration by harmony search algorithm. Expert Syst Appl 2012;39(10):9704–14.
[229] Jung DS, Kim CY. Finite element model updating on small-scale bridge model using the hybrid genetic algorithm. Struct Infrastruct Eng 2013;9:481–95. https://doi.org/10.1080/15732479.2011.564635.
[230] Shallan O, Eraky A, Sakr T, Khozam M. Structural Damage Detection using Genetic Algorithm by Static Measurements. Int J Trend Res Dev 2017;4:324–9.
[231] Shabbir F, Omenzetter P. Particle swarm optimization with sequential niche technique for dynamic finite element model updating. Comput Civ Infrastruct Eng 2015;30:359–75. https://doi.org/10.1111/mice.12100.
[232] Luo Z.,Yu L. PSO based Sparse Regularization Approach for Structural Damage Detection, in: 13th Int. Conf. Nat. Comput. Fuzzy Syst. Knowl. Discov. (ICNC-FSKD 2017), IEEE, 2017: pp. 1033–1039.
[233] Vakil Baghmisheh MT, Peimani M, Sadeghi MH, Ettefagh MM, Tabrizi AFA. hybrid particle swarm-Nelder-Mead optimization method for crack detection in cantilever beams. Appl Soft Comput J 2012;12:2217–26. https://doi.org/10.1016/j.asoc.2012.03.030.

[234] Saada MM, Arafa MH, Nassef AO. Finite element model updating approach to damage identification in beams using particle swarm optimization. Eng Optim 2013;45:677–96. https://doi.org/10.1080/0305215X.2012.704026.
[235] Kang F, Li JJ, Xu Q. Damage detection based on improved particle swarm optimization using vibration data. Appl Soft Comput J 2012;12:2329–35. https://doi.org/10.1016/j.asoc.2012.03.050.
[236] Li J, Chen J. Solving time-variant reliability-based design optimization by PSO-t-IRS: A methodology incorporating a particle swarm optimization algorithm and an enhanced instantaneous response surface. Reliab Eng Syst Saf 2019;191:106580.
[237] He RS, Hwang SF. Damage detection by a hybrid real-parameter genetic algorithm under the assistance of grey relation analysis. Eng Appl Artif Intell 2007;20:980–92. https://doi.org/10.1016/j.engappai.2006.11.020.
[238] Hwang SF, He RS. Improving real-parameter genetic algorithm with simulated annealing for engineering problems. Adv Eng Softw 2006;37:406–18. https://doi.org/10.1016/j.advengsoft.2005.08.002.
[239] Astroza R, Nguyen LT, Nestorović T. Finite element model updating using simulated annealing hybridized with unscented Kalman filter. Comput Struct 2016;177:176–91. https://doi.org/10.1016/j.compstruc.2016.09.001.
[240] Grafe H. Model Updating of Large Structural Dynamics Models Using Measured Response Functions. University of London 1998. https://doi.org/10.1002/eqe.2925.
[241] Jin SS, Jung HJ. Sequential surrogate modeling for efficient finite element model updating. Comput Struct 2016;168:30–45. https://doi.org/10.1016/j.compstruc.2016.02.005.
[242] Hemez FM, Doebling SW. Model validation and uncertainty quantification. Proc Int Modal Anal Conf - IMAC 2001;2:1153–8.
[243] Gunst RF, Mason RL. Fractional factorial design. Wiley Interdiscip Rev Comput Stat 2009;1:234–44. https://doi.org/10.1002/wics.27.
[244] Cheng QS, Koziel S, Bandler JW. Simplified space-mapping approach to enhancement of microwave device models. Int J RF and Microwave Comp Aid Eng 2006;16(5):518–35.
[245] Shahidi SG, Pakzad SN. Generalized Response Surface Model Updating Using Time Domain Data. J Struct Eng 2014;140:1–13. https://doi.org/10.1061/(asce)st.1943-541x.0000915.
[246] Queipo NV, Haftka RT, Shyy W, Goel T, Vaidyanathan R, Kevin TP. Surrogate-based analysis and optimization. Prog Aerosp Sci 2005;41:1–28. https://doi.org/10.1016/j.paerosci.2005.02.001.
[247] Wu J, Yan Q, Huang S, Zou C, Zhong J, Wang W. Finite Element Model Updating in Bridge Structures Using Kriging Model and Latin Hypercube Sampling Method. Adv Civ Eng 2018;2018:1–11.
[248] Gaspar B, Bucher C, Guedes SC. Reliability analysis of plate elements under uniaxial compression using an adaptive response surface approach. Ships Offshore Struct 2015;10:145–61. https://doi.org/10.1080/17445302.2014.912047.
[249] Mao J, Wang H, Li J. Bayesian Finite Element Model Updating of a Long-Span Suspension Bridge Utilizing Hybrid Monte Carlo Simulation and Kriging Predictor. KSCE J Civ Eng 2020;24:569–79. https://doi.org/10.1007/s12205-020-0983-4.
[250] Li J, Chen J, Wei J, Zhang X, Han B. Developing an Instantaneous Response Surface Method t-IRS for Time-Dependent Reliability Analysis. Acta Mech Solida Sin 2019;32:446–62. https://doi.org/10.1007/s10338-019-00096-5.
[251] Chaabane M, Mansouri M, Nounou H, Nounou M, Hamida A, Ben. Enhanced particle filter for states and parameters estimation in structural health monitoring applications. J Civ Struct Heal Monit 2016;6:521–43. https://doi.org/10.1007/s13349-016-0171-x.
[252] Zhou L, Wang L, Chen L, Ou J. Structural finite element model updating by using response surfaces and radial basis functions. Adv Struct Eng 2016;19:1446–62. https://doi.org/10.1177/1369433216643876.
[253] Ren W-X, Fang S-E, Deng M-Y. Response Surface-Based Finite-Element-Model Updating Using Structural Static Responses. J Eng Mech 2011;137:248–57. https://doi.org/10.1061/(asce)em.1943-7889.0000223.
[254] Zong Z, Lin X, Niu J. Finite element model validation of bridge based on structural health monitoring—Part I: Response surface-based finite element model updating. J Traffic Transp Eng (English Ed 2015;2:258–78. https://doi.org/10.1016/j.jtte.2015.06.001.
[255] Fang SE, Perera R. Damage identification by response surface based model updating using D-optimal design. Mech Syst Signal Process 2011;25:717–33. https://doi.org/10.1016/j.ymssp.2010.07.007.
[256] Conn AR, Scheinberg K, Vicente LN. Introduction to Derivative-Free Optimization. MPS-SIAM Series on. Optimization 2019.
[257] Zhou L, Yan G, Ou J. Response Surface Method Based on Radial Basis Functions for Modeling Large-Scale Structures in Model Updating. Comput Civ Infrastruct Eng 2013;28:210–26. https://doi.org/10.1111/j.1467-8667.2012.00803.x.
[258] Li X, Gong C, Gu L, Gao W, Jing Z, Su H. A sequential surrogate method for reliability analysis based on radial basis function. Struct Saf 2018;73:42–53. https://doi.org/10.1016/j.strusafe.2018.02.005.
[259] Liu Y, Li Y, Wang D, Zhang S. Model updating of complex structures using the combination of component mode synthesis and Kriging predictor. Sci World J 2014;2014:1–13.
[260] Simpson TW, Mauery TM, Korte JJ, Mistree F. Comparison of response surface and kriging models for multidisciplinary design optimization. 7th AIAA/USAF/NASA/ISSMO Symp Multidiscip Anal Optim 1998:381–91. https://doi.org/10.2514/6.1998-4755.
[261] Padil KH, Bakhary N, Hao H. The use of a non-probabilistic artificial neural network to consider uncertainties in vibration-based-damage detection. Mech Syst Signal Process 2017;83:194–209. https://doi.org/10.1016/j.ymssp.2016.06.007.
[262] Liu H, Song G, Jiao Y, Zhang P, Wang X. Damage identification of bridge based on modal flexibility and neural network improved by particle swarm optimization. Math Probl Eng 2014;2014:1–8.
[263] Tan ZX, Thambiratnam DP, Chan THT, Abdul RH. Detecting damage in steel beams using modal strain energy based damage index and Artificial Neural Network. Eng Fail Anal 2017;79:253–62. https://doi.org/10.1016/j.engfailanal.2017.04.035.
[264] Liu C-W, Huang X-H, Miao J-J, Ba G-Z. Modification of finite element models based on support vector machines for reinforced concrete beam vibrational analyses at elevated temperatures. Struct Control Heal Monit 2019;26(6):e2350.
[265] Yu W.,He H.,Zhang N. Finite element model updating based on least squares support vector machines, in: 6th Int. Symp. Neural Networks, ISNN 2009, Springer, Wuhan, China, 2009: pp. 296–303 https://doi.org/10.1007/978-3-642-01510-6_34.
[266] A. Jung Machine Learning: Fundations, Methodologies and Applications 1st editio, 2018 Springer Singapore 10.1007/978-981-16-8193-6.
[267] Teng J, Zhu Y, Zhou F, Li H, Ou J. Finite element model updating for large span spatial steel structure considering uncertainties. J Cent South Univ Technol 2010;17:857–62. https://doi.org/10.1007/s11771−010−0567−4.
[268] Badarinath PV, Chierichetti M, Kakhki FD. A machine learning approach as a surrogate for a finite element analysis: Status of research and application to one dimensional systems. Sensors 2021;21:1–18. https://doi.org/10.3390/s21051654.
[269] Zhu Y.,Zhang L. Finite element model updating based on least squares support vector machines. 2009. https://doi.org/10.1007/978-3-642-01510-6_34.
[270] Ivanova N, Gugleva V, Dobreva M, Pehlivanov I, Stefanov S, Andonova V. We are IntechOpen, the world ' s leading publisher of Open Access books Built by scientists, for scientists TOP 1 %. Intech 2016;i:13.
[271] Ben SM. Model selection and adaptive sampling in surrogate modeling. Kriging and beyond 2018.
[272] Forrester AIJ, Keane AJ. Recent advances in surrogate-based optimization. Prog Aerosp Sci 2009;45:50–79. https://doi.org/10.1016/j.paerosci.2008.11.001.
[273] Alexandrov N, Lewis RM. An Overview of First-Order Model Management for Engineering Optimization. Optim Eng 2001;2:413–30. https://doi.org/10.1023/A:1016042505922.
[274] Søndergaard J. Optimization using surrogate models-by space mapping technique. Technical University of Denmarka 2003.
[275] Chakraborty S, Sen A. Adaptive response surface based efficient Finite Element Model Updating. Finite Elem Anal Des 2014;80:33–40. https://doi.org/10.1016/j.finel.2013.11.002.
[276] Deng L, Cai CS. Bridge model updating using response surface method. Proc 12th Int Conf Eng Sci Constr Oper Challenging Environ - Earth Sp 2010:2311–20. https://doi.org/10.1061/41096(366)213.
[277] J. Han Y. Yang Theory and Implementation of Finite Element Model Updating of the Structures Based on Time Domain Data. DEStech Trans. Environ. Energy Earth Sci. 2016 https://doi.org/10.12783/dteees/peee2016/3948.
[278] Su L, Wan HP, Dong Y, Frangopol DM, Ling XZ. Efficient Uncertainty Quantification of Wharf Structures under Seismic Scenarios Using Gaussian Process Surrogate Model. J Earthq Eng 2021;25:117–38. https://doi.org/10.1080/13632469.2018.1507955.
[279] Shan D, Li Q, Khan I, Zhou X. A novel finite element model updating method based on substructure and response surface model. Eng Struct 2015;103:147–56. https://doi.org/10.1016/j.engstruct.2015.09.006.
[280] Moravej H, Chan THT, Jesus A, Nguyen K-D. Computation-Effective Structural Performance Assessment Using Gaussian Process-Based Finite Element Model Updating and Reliability Analysis. Int J Struct Stab Dyn 2020;20(10):2042003.
[281] Bucher CG, Bourgund U. A fast and efficient response surface approach for structural reliability problems. Struct Saf 1990;7:57–66. https://doi.org/10.1016/0167-4730(90)90012-E.
[282] Das PK, Zheng Y. Cumulative formation of response surface and its use in reliability analysis. Probabilistic Eng Mech 2000;15:309–15. https://doi.org/10.1016/S0266-8920(99)00030-2.
[283] Rutherford AC, Inman DJ, Park G, Hemez FM. Use of response surface metamodels for identification of stiffness and damping coefficients in a simple dynamic system. Shock Vib 2005;12:317–31. https://doi.org/10.1155/2005/484283.
[284] Dey P, Talukdar S, Bordoloi DJ. Multiple-crack identification in a channel section steel beam using a combined response surface methodology and genetic algorithm: Multiple-Crack Identification. Struct Control Health Monit 2016;23(6):938–59.
[285] Grabec I. Biological Cybemetics. Neural Networks 1990;409:403–9.
[286] Daqi G, Shouyi W. An optimization method for the topological structures of feed-forward multi-layer neural networks. Pattern Recognit 1998;31:1337–42. https://doi.org/10.1016/S0031-3203(97)00160-X.
[287] Yuen KV, Ortiz GA. Multiresolution Bayesian nonparametric general regression for structural model updating. Struct Control Heal Monit 2018;25:1–14. https://doi.org/10.1002/stc.2077.
[288] Saraygord AS, Enayatollahi F, Xu X, Liang X. Machine learning-based methods in structural reliability analysis: A review. Reliab Eng Syst Saf 2022;219:108223. https://doi.org/10.1016/j.ress.2021.108223.
[289] Yin T, Zhu HP. An efficient algorithm for architecture design of Bayesian neural network in structural model updating. Comput Civ Infrastruct Eng 2020;35:354–72. https://doi.org/10.1111/mice.12492.

[290] Карускевич М.,Корчук О.,Лісовська М. Structural Health monitoring, in: A. Wicks (Ed.), A Conf. Expo. Struct. Dyn., Springer, 2013: pp. 117–122.
[291] S.J.S. Hakim R.H. Abdul Frequency response function-based structural damage identification using artificial neural networks-A review Res. J. Appl. Sci. Eng. Technol. 7 2014 1750 1764 https://doi.org/10.19026/rjaset.7.459.
[292] Meruane V, Mahu J. Real-time structural damage assessment using artificial neural networks and antiresonant frequencies. Shock Vib 2014;2014:1–14.
[293] Z. Li D. Feng M.Q. Feng X. Xu System identification of the suspension tower of Runyang Bridge based on ambient vibration tests Smart Struct. Syst. 19 2017 523 538 https://doi.org/10.12989/sss.2017.19.5.523.
[294] Ponsi F.,Bassoli E.,Vincenzi L. Bayesian Model Updating and Parameter Uncertainty Analysis of a Damaged Fortress Through Dynamic Experimental Data, in: Civ. Struct. Heal. Monit. CSHM 2021. Lect. Notes Civ. Eng., Springer, 2021: pp. 515–533 https://doi.org/https://doi.org/10.1007/978-3-030-74258-4_34.
[295] Fujita K, Takewaki I. Stiffness identification of high-rise buildings based on statistical model-updating approach. Front Built Environ 2018;4:1–11. https://doi.org/10.3389/fbuil.2018.00009.
[296] Hu J, Yang JH. Operational Modal Analysis and Bayesian Model Updating of a Coupled Building. Int J Struct Stab Dyn 2019;19:1–15. https://doi.org/10.1142/S0219455419400121.
[297] Lam HF, Hu J, Adeagbo MO. Bayesian model updating of a 20-story office building utilizing operational modal analysis results. Adv Struct Eng 2019;22:3385–94. https://doi.org/10.1177/1369433218825043.
[298] Simoen E, Papadimitriou C, Lombaert G. On prediction error correlation in Bayesian model updating. J Sound Vib 2013;332:4136–52. https://doi.org/10.1016/j.jsv.2013.03.019.
[299] Goller B, Schuëller GI. Investigation of model uncertainties in Bayesian structural model updating. J Sound Vib 2011;330:6122–36. https://doi.org/10.1016/j.jsv.2011.07.036.
[300] C. Argyris C. Papdimitriou P. Panetsos P. Tsopelas Bayesian Model Updating Using Features of Modal Data: Application to the Metsovo Bridge J. Sensors Actuator Networks 9 2020 https://doi.org/doi:10.3390/jsan9020027.
[301] Sun H, Büyüköztürk O. Bayesian model updating using incomplete modal data without mode matching. Heal Monit Struct Biol Syst 2016;2016(9805):98050D. https://doi.org/10.1117/12.2219300.
[302] Cheung SH, Bansal S. A new Gibbs sampling based algorithm for Bayesian model updating with incomplete complex modal data. Mech Syst Signal Process 2017;92:156–72. https://doi.org/10.1016/j.ymssp.2017.01.015.
[303] Akhlaghi MM, Bose S, Moaveni B, Stavridis A. Bayesian model updating of a damaged school building in Sankhu. Nepal Conf Proc Soc Exp Mech Ser 2019;3:235–44. https://doi.org/10.1007/978-3-319-74793-4_28.
[304] Kernicky T, Whelan M, Al-Shaer E. Vibration-based damage detection with uncertainty quantification by structural identification using nonlinear constraint satisfaction with interval arithmetic. Struct Heal Monit 2019;18:1569–89. https://doi.org/10.1177/1475921718806476.
[305] Moravej H, Chan THT, Nguyen KD, Jesus A. Vibration-based Bayesian model updating of civil engineering structures applying Gaussian process metamodel. Adv Struct Eng 2019;22:3487–502. https://doi.org/10.1177/1369433219858723.
[306] Hou J, An Y, Wang S, Wang Z, Jankowski Ł, Ou J. Structural Damage Localization and Quantification Based on Additional Virtual Masses and Bayesian Theory. J Eng Mech 2018;144:04018097. https://doi.org/10.1061/(asce)em.1943-7889.0001523.
[307] Astroza R, Ebrahimian H, Conte JP. Batch and recursive bayesian estimation methods for nonlinear structural system identification. Springer Ser Reliab Eng 2017:341–64. https://doi.org/10.1007/978-3-319-52425-2_15.
[308] Ebrahimian H, Astroza R, Conte JP, de Callafon RA. Nonlinear finite element model updating for damage identification of civil structures using batch Bayesian estimation. Mech Syst Signal Process 2017;84:194–222. https://doi.org/10.1016/j.ymssp.2016.02.002.
[309] Ng CT. Bayesian model updating approach for experimental identification of damage in beams using guided waves. Struct Heal Monit 2014;13:359–73. https://doi.org/10.1177/1475921714532990.
[310] Serhat EY, Gundes BP. Inverse propagation of uncertainties in finite element model updating through use of fuzzy arithmetic. Eng Appl Artif Intell 2013;26:357–67. https://doi.org/10.1016/j.engappai.2012.10.003.
[311] Dominik I, Iwaniec M, Lech Ł. Low frequency damage analysis of electric pylon model by fuzzy logic application. J. Low Freq. Noise Vib. Act. Control 2013;32:239–51. https://doi.org/10.1260/0263-0923.32.3.239.
[312] Mojtahedi A, Hokmabady H, Abyaneh SSZ, Nassiraei H. Establishment of a hybrid Fuzzy-Krill Herd approach for novelty detection applied to damage classification of offshore jacket-type structures. J Mar Sci Technol 2019;24:812–29. https://doi.org/10.1007/s00773-018-0589-4.
[313] Moens D, Vandepitte D. A fuzzy finite element procedure for the calculation of uncertain frequency-response functions of damped structures: Part 1 - Procedure. J Sound Vib 2005;288:431–62. https://doi.org/10.1016/j.jsv.2005.07.001.
[314] De Gersem H, Moens D, Desmet W, Vandepitte D. A fuzzy finite element procedure for the calculation of uncertain frequency response functions of damped structures: Part 2 - Numerical case studies. J Sound Vib 2005;288:463–86. https://doi.org/10.1016/j.jsv.2005.07.002.
[315] Tibshirani R. Regression shrinkage and selection via the lasso: a retrospective. J R Stat Soc 2011;73:273–82.
[316] Yao H, Gerstoft P, Shearer PM, Mecklenbräuker C. Compressive sensing of the Tohoku-Oki Mw 9.0 earthquake: Frequency-dependent rupture modes. Geophys Res Lett 2011;38:1–5. https://doi.org/10.1029/2011GL049223.
[317] Hernandez EM. Identification of isolated structural damage from incomplete spectrum information using l1-norm minimization. Mech Syst Signal Process 2014;46:59–69. https://doi.org/10.1016/j.ymssp.2013.12.009.
[318] Ahmadian H, Mottershead JE, Friswell MI. Regularisation methods for finite element model updating. Mech Syst Signal Process 1998;12:47–64. https://doi.org/10.1006/mssp.1996.0133.
[319] Gutknecht MH. A Brief Introduction to Krylov Space Methods for Solving Linear Systems. Front Comput Sci 2007:53–62. https://doi.org/10.1007/978-3-540-46375-7_5.
[320] Zhou XQ, Xia Y, Weng S. L1 regularization approach to structural damage detection using frequency data. Struct Heal Monit 2015;14:571–82. https://doi.org/10.1177/1475921715604386.
[321] Hou R, Xia Y, Zhou X. Structural damage detection based on l1 regularization using natural frequencies and mode shapes. Struct Control Heal Monit 2018;25:1–17. https://doi.org/10.1002/stc.2107.
[322] A. Garcia-Palencia E. Santini-Bell M. Gul N. Catbas A FRF-based algorithm for damage detection using experimentally collected data Struct. Monit. Maint. 2 2015 399 418 https://doi.org/10.12989/smm.2015.2.4.399.
[323] Zhang* C.D., Xu Y.L. Comparative studies on damage identification with Tikhonov regularization and sparse regularization Struct. Control Heal. Monit. 23 2015 560 579 https://doi.org/doi.org/10.1002/stc.1785.
[324] Pan C, Yu L. Sparse regularization-based damage detection in a bridge subjected to unknown moving forces. J Civ Struct Heal Monit 2019;9:425–38. https://doi.org/10.1007/s13349-019-00343-w.
[325] Ding Z, Li J, Hao H. Non-probabilistic method to consider uncertainties in structural damage identification based on Hybrid Jaya and Tree Seeds Algorithm. Eng Struct 2020;220:110925. https://doi.org/10.1016/j.engstruct.2020.110925.
[326] Hua XG, Ni YQ, Ko JM. Adaptive regularization parameter optimization in output-error-based finite element model updating. Mech Syst Signal Process 2009;23:563–79. https://doi.org/10.1016/j.ymssp.2008.05.002.

Further Reading

[162] Alkayem NF, Cao M, Zhang Y, Bayat M, Su Z. Structural damage detection using finite element model updating with evolutionary algorithms: a survey. Neural Comput Appl 2018;30:389–411. https://doi.org/10.1007/s00521-017-3284-1.
[163] Wang D, Tan Z, Li Y, Liu Y. Review of the application of finite element model updating to civil structures. Key Eng Mater 2014;574:107–15. https://doi.org/10.4028/www.scientific.net/KEM.574.107.