

# Inequality of Hardy-type for n-convex function via interpolation polynomial and Green functions

---

**Pokaz, Dora**

*Source / Izvornik:* **Mathematical inequalities & applications, 2023, -, - - -**

**Journal article, Published version**

**Rad u časopisu, Objavljena verzija rada (izdavačev PDF)**

*Permanent link / Trajna poveznica:* <https://um.nsk.hr/um:nbn:hr:237:088295>

*Rights / Prava:* [In copyright](#)/[Zaštićeno autorskim pravom.](#)

*Download date / Datum preuzimanja:* **2025-02-28**

*Repository / Repozitorij:*

[Repository of the Faculty of Civil Engineering,  
University of Zagreb](#)



See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/375884054>

# Inequality of Hardy-type for $n$ -convex function via interpolation polynomial and Green functions

Article in *Mathematical Inequalities & Applications* - January 2023

DOI: 10.7153/mia-2023-26-59

---

CITATION

1

READS

15

1 author:



Dora Pokaz

University of Zagreb

27 PUBLICATIONS 44 CITATIONS

SEE PROFILE

## INEQUALITY OF HARDY-TYPE FOR $n$ -CONVEX FUNCTION VIA INTERPOLATION POLYNOMIAL AND GREEN FUNCTIONS

DORA POKAZ

**Abstract.** We obtain new results on the Hardy-type inequality in the general context, in terms of measure spaces with positive  $\sigma$ -finite measures. The connection is made between the difference operator derived from the Hardy-type inequality on the one hand and the expression containing the interpolating polynomial of Abel-Gontscharoff and the four Green functions on the other hand. We discuss the  $n$ -convexity of the function and consider the result depending on the parity of the indexes  $n$  and  $m$ . Further results are obtained by using the Hölder inequality for conjugate exponents  $p$  and  $q$ . Finally, we derive upper bounds for the remainder, obtained from the main result, using the Čebyšev functional. The Ostrowski-type bound for the generalized Hardy inequality is also given.

**Mathematics subject classification (2020):** 26D10, 26D15, 39B62.

**Keywords and phrases:** Convex function, Hardy-type inequality, Abel-Gontscharoff interpolating polynomial, Green function, Čebyšev functional.

### REFERENCES

- [1] R. P. AGARWAL AND P. J. Y. WONG, *Error Inequalities in Polynomial Interpolation and their Applications*, Kluwer Academic Publishers, Dordrecht, 1993.
- [2] A. AGLIĆ ALJINOVIĆ, J. PEČARIĆ, AND A. VUKELIĆ, *On some Ostrowski type inequalities via Montgomery identity and Taylor's formula II*, Tamkang Jour. Math. **36** (4) (2005), 279–301.
- [3] P. CERONE, S. S. DRAGOMIR, *Some new Ostrowski-type bounds for the Cebyshev functional and applications*, J. Math. Inequal. **8** (1) (2014), 159–170.
- [4] P. J. DAVIS, *Interpolation and Approximation*, Blaisdell, Boston, 1961.
- [5] V. L. GONTSCHAROFF, *Theory of Interpolation and Approximation of Functions*, Gostekhizdat, Moscow, 1954.
- [6] G. H. HARDY, *Notes on some points in the integral calculus LX: An inequality between integrals* (60), Messenger of Math. **54** (1925), 150–156.
- [7] S. KAIJSER, L. NIKOLOVA, L. E. PERSSON AND A. WEDESTIG, *Hardy-Type Inequalities via Convexity*, Math. Inequal. Appl., **8** (2005), 403–417.
- [8] K. KRULIĆ HIMMELREICH, J. PEČARIĆ, D. POKAZ, *Inequalities of Hardy and Jensen*, Element, Zagreb, 2013.
- [9] K. KRULIĆ HIMMELREICH, J. PEČARIĆ, D. POKAZ, M. PRALJAK, *Generalizations of Hardy-Type Inequalities by Montgomery Identity and New Green Functions*, Axioms 2023, **12**, 434, 1–13.
- [10] K. KRULIĆ HIMMELREICH, J. PEČARIĆ, D. POKAZ, D. AND M. PRALJAK, *Generalizations of Hardy Type Inequalities by Abel-Gontscharoff's Interpolating Polynomial*, Mathematics 2021, **9**, 1724, <https://doi.org/10.3390/math9151724>.
- [11] A. KUFNER, L. MALIGRANDA, AND L.-E. PERSSON, *The Hardy Inequality. About its History and Some Related Results*, Vydavatelský Servis Publishing House, Pilsen, 2007.
- [12] A. KUFNER, L. MALIGRANDA, AND L.-E. PERSSON, *The prehistory of the Hardy inequality*, Amer. Math. Monthly **113** (2006), no. **8**, 715–732.
- [13] J. E. PEČARIĆ, F. PROSCHAN, AND Y. L. TONG, *Convex functions, Partial Orderings and Statistical Applications*, Academic Press, San Diego, 1992.

- [14] J. M. WHITTAKER, *Interpolation Function Theory*, Cambridge Univ. Press, Cambridge, England (1935).
- [15] D. V. WIDDER, *The Laplace Transform*, Princeton Univ. Press, New Jersey (1941).