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INEQUALITY OF HARDY–TYPE FOR n–CONVEX FUNCTION VIA INTERPOLATION POLYNOMIAL AND GREEN FUNCTIONS

DORA POKAZ

Abstract. We obtain new results on the Hardy-type inequality in the general context, in terms of measure spaces with positive σ-finite measures. The connection is made between the difference operator derived from the Hardy-type inequality on the one hand and the expression containing the interpolating polynomial of Abel-Gontscharoff and the four Green functions on the other hand. We discuss the n-convexity of the function and consider the result depending on the parity of the indexes n and m. Further results are obtained by using the Hölder inequality for conjugate exponents p and q. Finally, we derive upper bounds for the remainder, obtained from the main result, using the Čebyšev functional. The Ostrowski-type bound for the generalized Hardy inequality is also given.

Mathematics subject classification (2020): 26D10, 26D15, 39B62.
Keywords and phrases: Convex function, Hardy-type inequality, Abel-Gontscharoff interpolating polynomial, Green function, Čebyšev functional.

REFERENCES

[1] R. P. AGARWAL AND P. J. Y. WONG, *Error Inequalities in Polynomial Interpolation and their Applications*, Kluwer Academic Publishers, Dordrecht, 1993.
[2] A. AGLIĆ ALJINOVIĆ, J. PEČARIĆ, AND A. VUKELIĆ, *On some Ostrowski type inequalities via Montgomery identity and Taylor's formula II*, Tamkang Jour. Math. **36** (4) (2005), 279–301.
[3] P. CERONE, S. S. DRAGOMIR, *Some new Ostrowski-type bounds for the Cebysev functional and applications*, J. Math. Inequal. **8** (1) (2014), 159–170.
[4] P. J. DAVIS, *Interpolation and Approximation*, Blaisdell, Boston, 1961.
[5] V. L. GONTSCHAROFF, *Theory of Interpolation and Approximation of Functions*, Gostekhizdat, Moscow, 1954.
[6] G. H. HARDY, *Notes on some points in the integral calculus LX: An inequality between integrals* (60), Messenger of Math. **54** (1925), 150–156.
[7] S. KAIJSER, L. NIKOLOVA, L. E. PERSSON AND A. WEDESTIG, *Hardy-Type Inequalities via Convexity*, Math. Inequal. Appl., **8** (2005), 403–417.
[8] K. KRULIĆ HIMMELREICH, J. PEČARIĆ, D. POKAZ, *Inequalities of Hardy and Jensen*, Element, Zagreb, 2013.
[9] K. KRULIĆ HIMMELREICH, J. PEČARIĆ, D. POKAZ, M. PRALJAK, *Generalizations of Hardy-Type Inequalities by Montgomery Identity and New Green Functions*, Axioms 2023, **12**, 434, 1–13.
[10] K. KRULIĆ HIMMELREICH, J. PEČARIĆ, D. POKAZ, D. AND M. PRALJAK, *Generalizations of Hardy Type Inequalities by Abel-Gontscharoff's Interpolating Polynomial*, Mathematics 2021, 9, 1724, https://doi.org/10.3390/math9151724.
[11] A. KUFNER, L. MALIGRANDA, AND L.-E. PERSSON, *The Hardy Inequality. About its History and Some Related Results*, Vydavatelsky Servis Publishing House, Pilsen, 2007.
[12] A. KUFNER, L. MALIGRANDA, AND L.-E. PERSSON, *The prehistory of the Hardy inequality*, Amer. Math. Monthly **113** (2006), no. **8**, 715–732.
[13] J. E. PEČARIĆ, F. PROSCHAN, AND Y. L. TONG, *Convex functions, Partial Orderings and Statistical Applications*, Academic Press, San Diego, 1992.

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[14] J. M. WHITTAKER, *Interpolation Function Theory*, Cambridge Univ. Press, Cambridge, England (1935).
[15] D. V. WIDDER, *The Laplace Transform*, Princeton Univ. Press, New Jersey (1941).